Extending Opentype math

—

Making decisions

Mikael Sundqvist

with Hans Hagen

Illustrations: Sofia Kockum

Why? Many OpenType math fonts exist (Ulrik's talk), but no standard. “It is a mess.”

What? We present some conclusions. The presentation will be visual, not so technical.

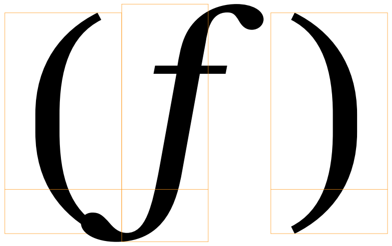
Goal? Simple general solutions that work with most fonts, no hacks required.

Who? We aim at ConTEXt users.

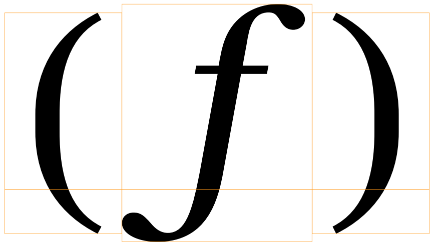
Context? This is part of a bigger math project.

PRESENT AND FUTURE TEAM MEMBERS

AVOID OVERLAPPING DUE TO BAD BOUNDING BOXES

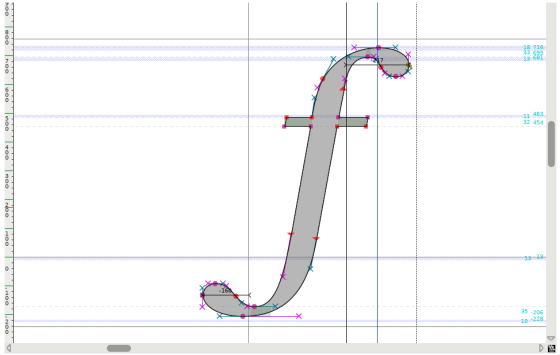


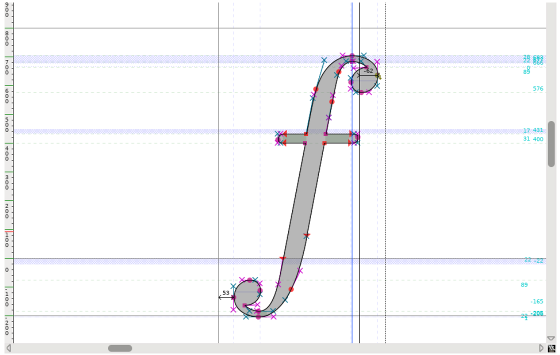
before

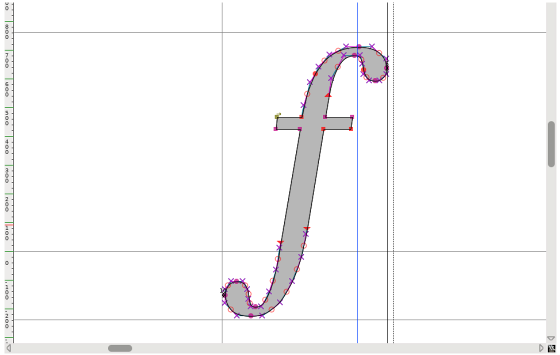


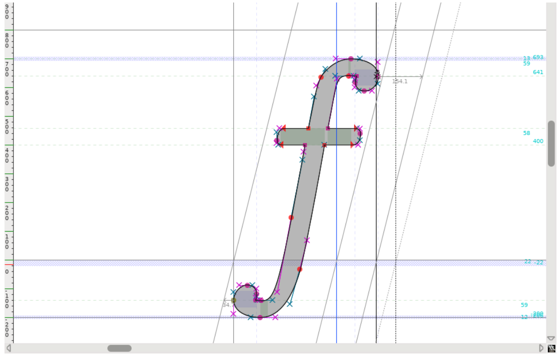
after

BOUNDING BOXES–FONTS ARE DIFFERENT


bonum


latin modern


stix two


concrete

BOUNDING BOXES, WHAT DO WE DO?

- Add italic correction to the width.
- Add right bottom kern instead.
- Ensure that the glyph does not stick out to the left.

BONUM USES A LOT OF ITALIC CORRECTION

NO CHANGE FOR SUB/SUPERSCRIPTS

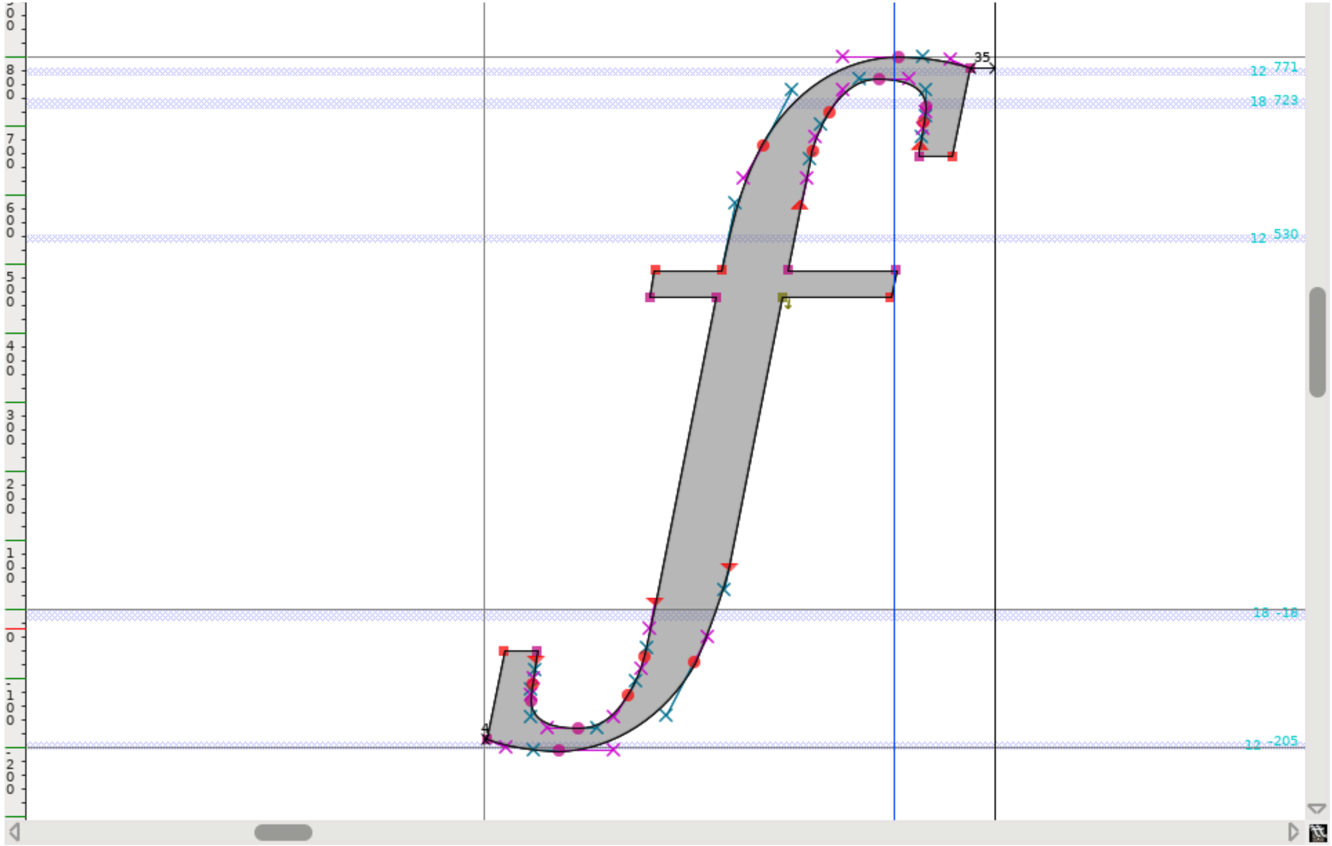
$$f_0^1$$

before

$$f_0^1$$

after

LUCIDA USES KERNS INSTEAD OF ITALIC CORRECTION



NO CHANGE FOR SUB/SUPERSCRIPTS

$$f_0^1$$

before

$$f_0^1$$

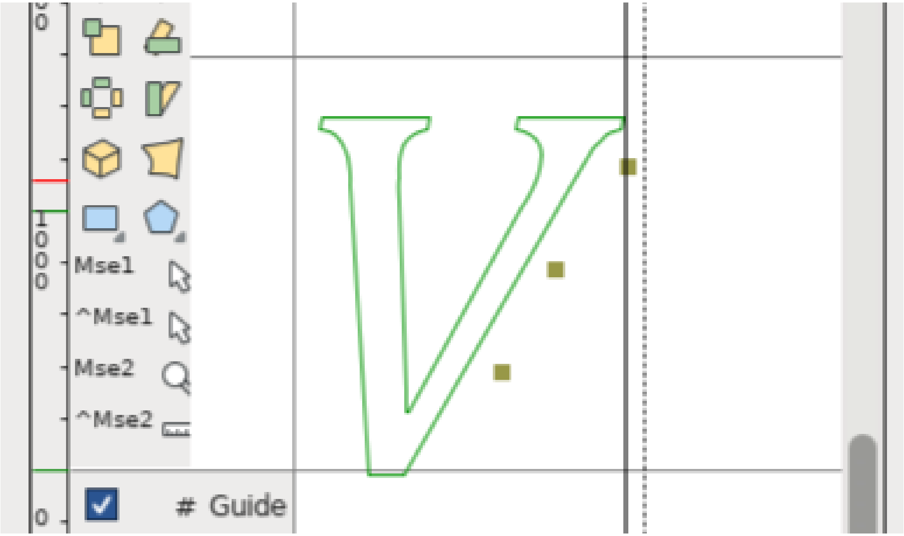
after

ITALIC CORRECTION, WHAT DO WE DO?

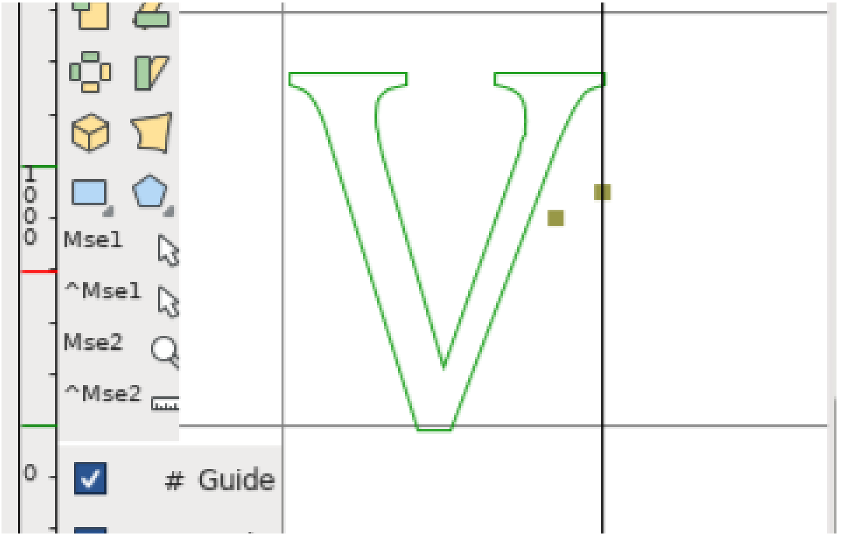
- Italic correction transformed to corner kerns.
- Large operators: if nolimits, then italic correction is converted as for other characters.
- Large operators: if limits, then top and bottom anchoring.

  Italic correction is not used (relevant) after that.

STAIRCASE KERNS ARE UNRELIABLE



italic V

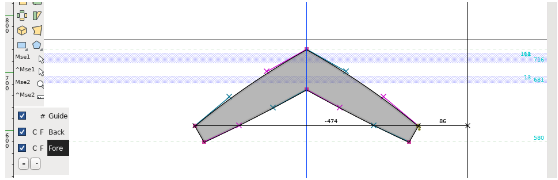


upright V

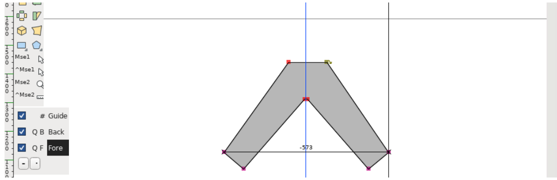
STAIRCASE KERNS, WHAT DO WE DO?

- We transform staircase kerns into corner kerns.
- Use the lowest point; assume it works.

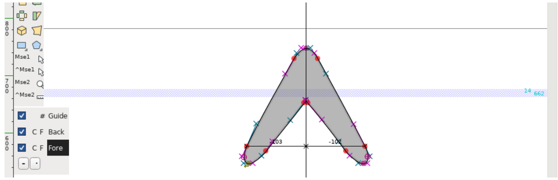
ACCENTS ARE SHIFTED TO THE LEFT, NO WIDTH



bonum

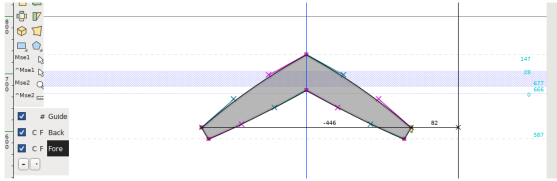


cambria

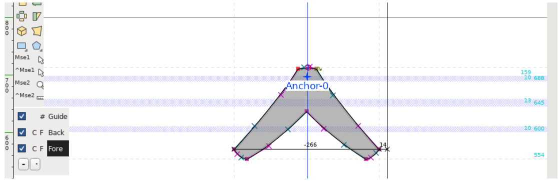


garamond math

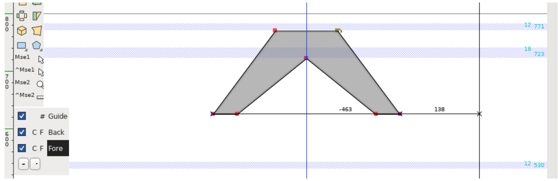
kp math



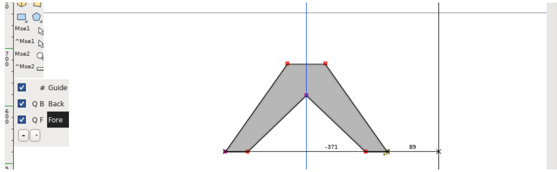
latin modern



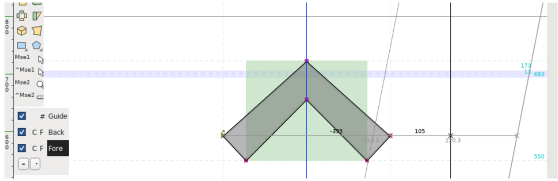
libertinus



lucida



stix two



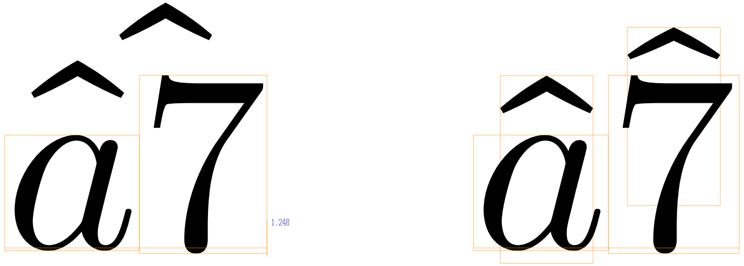
xcharter

ACCENTS DESERVE THEIR SPACE

$$\vec{p'}$$

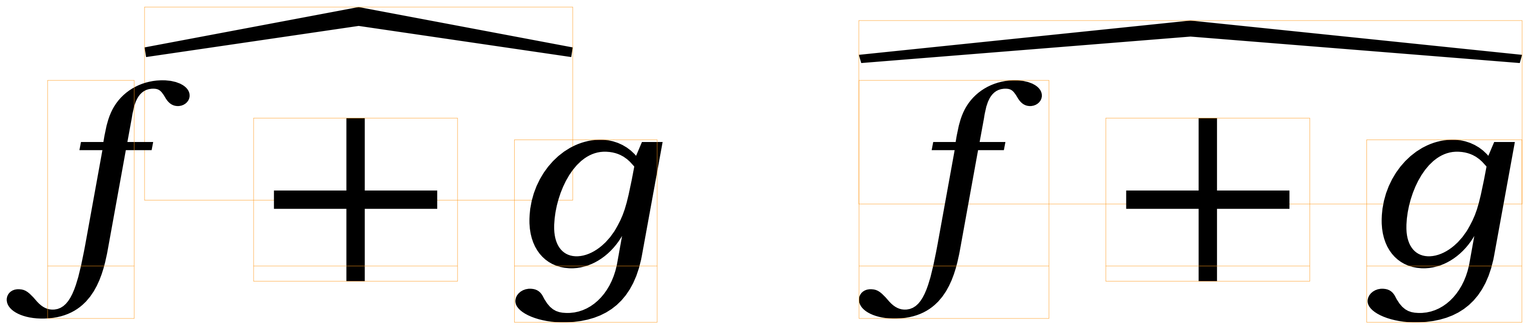
$$\vec{p}\,'$$

SOME ANCHOR POINTS NEED TO BE CORRECTED



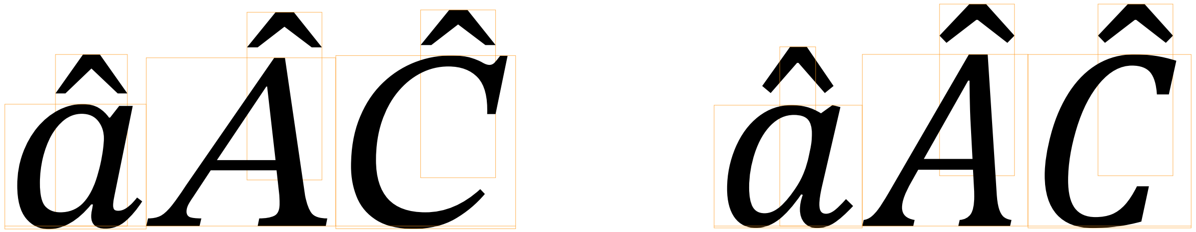
before after

SOME ACCENTS CAN BE SCALED



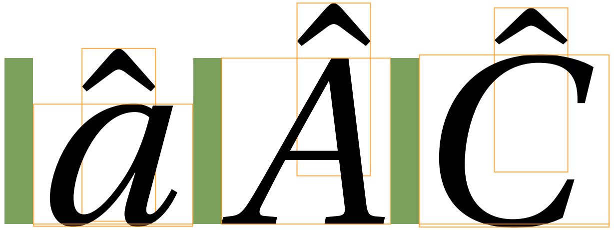
before after

FLATTENED ACCENTS REDUCE THE RISK OF LINE SPREAD

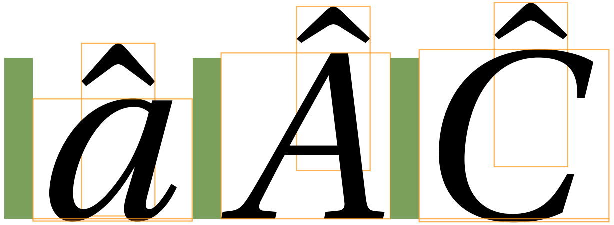


stix two cambria

WE CAN FAKE FLATTENED ACCENTS



erewhon before

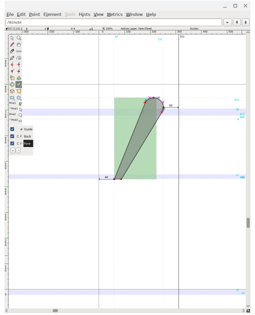


erewhon after

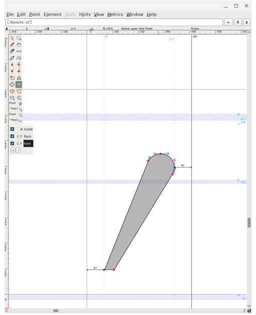
ACCENTS, WHAT DO WE DO?

- Give all accents a proper bounding box.
- Set anchor points in the middle for some alphabets.
- Stretch and shrink accents.
- Emulate flattened accents.
- Provide support for underaccents.

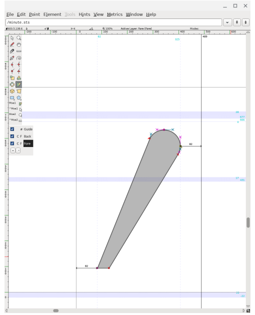
PRIMES ARE DONE DIFFERENT IN DIFFERENT FONTS



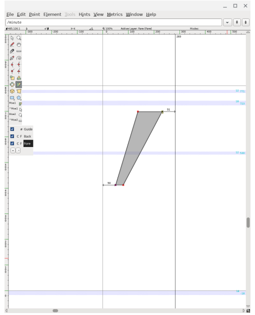
lm



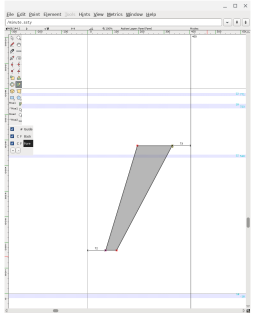
st



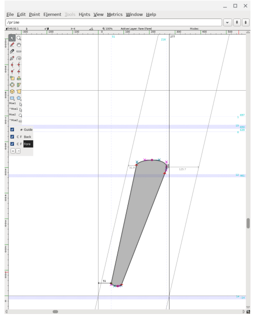
sts



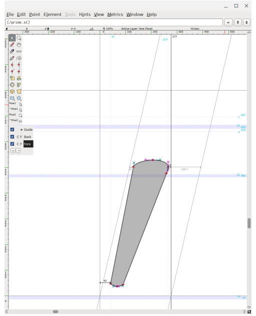
lucida



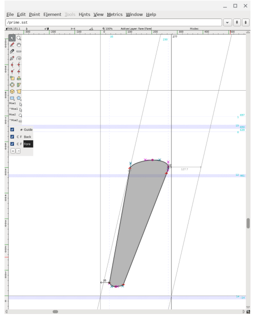
ssty



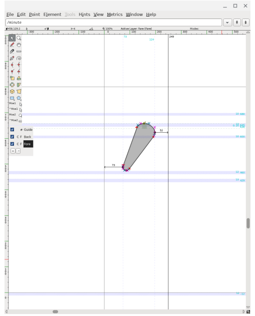
erewhon



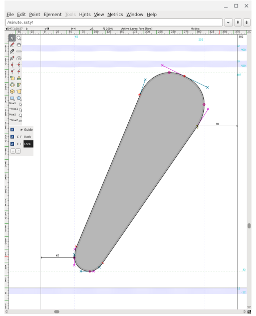
st



sst



libertinus



ssty1

PRIMES, WHAT DO WE DO?

- Tweak in goodie files.
- Primes always go after superscripts.

RULES ARE BORING AND RECTANGULAR

```
\switchtobodyfont[modern-nt]
\startformula
  \setmscale{4000}
  \sqrt{\frac{1+x}{1-x}}
\stopformula
```

$$\sqrt{\frac{1+x}{1-x}}$$

NO RULES – SHOW INSTEAD THE SPIRIT OF THE FONT

```
\switchtobodyfont[modern]
\startformula
  \setmscale{4000}
  \sqrt{\frac{1+x}{1-x}}
\stopformula
```

$$\sqrt{\frac{1+x}{1-x}}$$

ANTYKWA TORUŃSKA HAS A BEAUTIFUL SPIRIT

```
\switchtobodyfont[antykwa]
\startformula
  \setmscale{4000}
  \sqrt{\frac{1+x}{1-x}}
\stopformula
```

$$\sqrt{\frac{1+x}{1-x}}$$

NO RULES, REALLY?

- Indeed, no rules.
- Use glyphs instead (if they exist).

MORE ATOM CLASSES ENABLE A MORE FLEXIBLE INTERATOM SPACING

$$3a + \sqrt{2} + 3\int_0^1 f(x)\,dx = 3\frac{a}{b}$$

ordbin binrad radbin bindig digope opeord clodif ordrel reldig digfra

SPACING, WHAT DO WE DO?

- Introduce and set up several new atom classes.
- Make some old auto-inserted spaces configurable.

$$h^3 + h_2 + h_2^3 + h'$$

$$h^3 + h_2 + h_2^3 + h'$$

$$h^3 + h_2 + h_2^3 + h'$$

FONT PARAMETERS, WHAT DO WE DO?

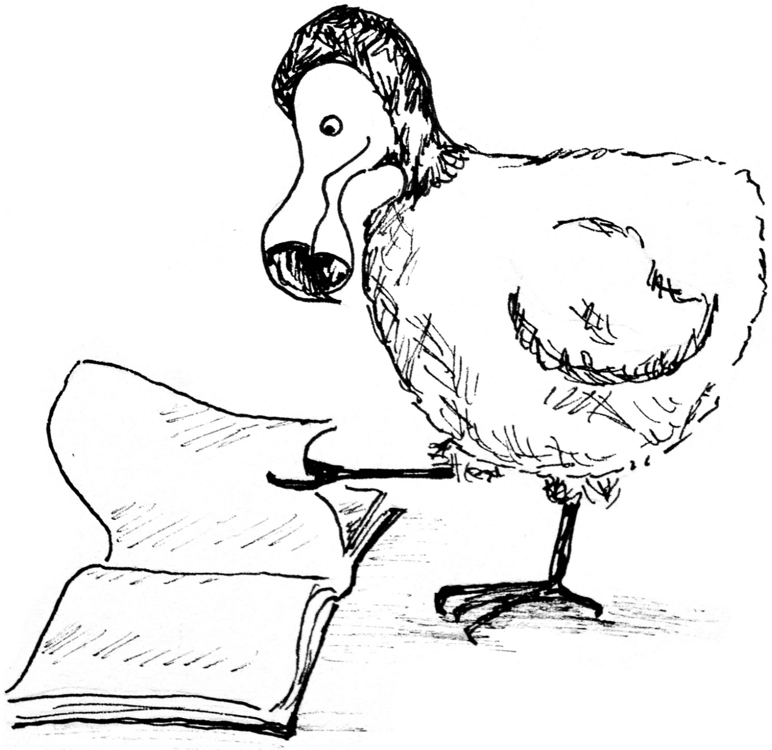
- We use the ones that make sense; many don't.
- We add a few. This extends OpenType math.
- We tweak them in the goodie files.

REMOVE UNWANTED SPACES

$$f(x) \neq f_1(x) \neq f_{x_2}(x) \neq f_{x_2}\, \left(\frac{1}{x}\right)$$

$$f(x) \neq f_1(x) \neq f_{x_2}(x) \neq f_{x_2}\left(\frac{1}{x}\right)$$

WHAT IS NEXT?



BREAKING DISPLAYED FORMULAS, PENALTY SETUP NEEDED

Long formula

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{199} - \frac{1}{200} = \frac{1}{101} + \frac{1}{102} + \dots + \frac{1}{200}$$

Automatically broken

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{199} - \frac{1}{200} =$$
$$\frac{1}{101} + \frac{1}{102} + \dots + \frac{1}{200}$$

Breakpoint by setting a penalty

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{199} - \frac{1}{200}$$
$$= \frac{1}{101} + \frac{1}{102} + \dots + \frac{1}{200}$$

MULTIPLE PASSES CAN IMPROVE LINE BREAKING

Three straight lines parallel to the sides of a triangle meet at one point, the sides of the triangle cutting off the line segments of length x each. Find x if the lengths of the triangle's sides are a, b and c.

1

Three straight lines parallel to the sides of a triangle meet at one point, the sides of the triangle cutting off the line segments of length x each. Find x if the lengths of the triangle's sides are a, b and c.

2

Three straight lines parallel to the sides of a triangle meet at one point, the sides of the triangle cutting off the line segments of length x each. Find x if the lengths of the triangle's sides are a, b and c.

3

1. Nothing special.
2. `\preshortinlinepenalty10000`
3. `\preshortinlinepenalty10000` and multiple passes.

A MULTIPASS SETUP (ASK FOR DETAILS)

```
\startsetups align:pass:tug
    \pretolerance 100
    \tolerance    200
    \parpasses    2
        classes                \indecentparpassclasses
        tolerance              300
        extrahyphenpenalty      50
    next
        threshold              0.025pt
        tolerance              350
        adjustspacing            3
        adjustspacingstep        1
        adjustspacingshrink     20
        adjustspacingstretch    40
        emergencystretch       .25\bodyfontsize
    \relax
\stopsetups
```

EXTRA PENALTIES AT THE BEGINNING AND END OF FORMULAS CAN IMPROVE LINE BREAKS

Clearly $1+2=3$. It is amusing to find that $4+5+6=7+8$ and perhaps very surprising that $9+10+11+12=13+14+15$. Check that $16+17+18+19+20=21+22+23+24$, and show that this pattern continues *ad infinitum*.

1

Clearly $1+2=3$. It is amusing to find that $4+5+6=7+8$ and perhaps very surprising that $9+10+11+12=13+14+15$. Check that $16+17+18+19+20=21+22+23+24$, and show that this pattern continues *ad infinitum*.

2

1. Nothing special.
2. Here we used

```
\setmathpostpenalty\mathbinarycode600
\setmathpostpenalty\mathrelationcode400
\mathforwardpenalties 3 200 100 50
\mathbackwardpenalties 3 200 100 50
```

IT IS POSSIBLE TO PEEK INTO THE PARBUILDER

15. (This procedure maintains four integers (A, B, C, D) with the invari- [1] b=89 d=32.35059pt p=5.414 r=0.966 (dt=90) (0pt)
ant meaning that "our remaining job is to output the continued fraction for [2] b=3 d=9.34784pt p=1.564 r=0.304 (dt=3) (0pt)
$(Ay + B)/(Cy + D)$, where y is the input yet to come.") Initially set $j \leftarrow$ [3] b=190 d=47.8333pt p=8.005 r=1.240 (dt=191) (0pt)
$k \leftarrow 0$, $(A, B, C, D) \leftarrow (a, b, c, d)$; then input x_j and set $(A, B, C, D) \leftarrow$ [4] b=103 d=49.7424pt p=8.325 r=1.012 (dt=104) (0pt)
$(Ax_j + B, A, Cx_j + D, C)$, $j \leftarrow j + 1$, one or more times until $C + D$ has the [5] b=5 d=17.98949pt p=3.011 r=0.366 (dt=5) (0pt)
same sign as C. (When $j > 1$ and the input has not terminated, we know that [6] b=7 d=-9.79172pt p=-1.639 r=0.405 (dt=7) (0pt)
$1 < y < \infty$; and when $C + D$ has the same sign as C we know therefore that [7] b=1 d=8.76291pt p=1.467 r=0.180 (dt=1) (0pt)
$(Ay + B)/(Cy + D)$ lies between $(A + B)/(C + D)$ and A/C.) Now comes the [8] b=4 d=13.42082pt p=2.246 r=0.337 (dt=4) (0pt)
general step: If no integer lies strictly between $(A + B)/(C + D)$ and A/C, out- [9] b=3 d=-7.34885pt p=-1.230 r=0.298 (dt=3) (0pt)
put $X_k \leftarrow \lfloor A/C \rfloor$, and set $(A, B, C, D) \leftarrow (C, D, A - X_k C, B - X_k D)$, $k \leftarrow k + 1$; [10] b=0 d=-5.07207pt p=-0.849 r=0.147 (dt=0) (0pt)
otherwise input x_j and set $(A, B, C, D) \leftarrow (Ax_j + B, A, Cx_j + D, C)$, $j \leftarrow j + 1$. [11] b=1 d=-7.32619pt p=-1.226 r=0.211 (dt=1) (0pt)
The general step is repeated ad infinitum. However, if at any time the *final* x_j [12] b=2 d=-5.82181pt p=-0.974 r=0.260 (dt=2) (0pt)
is input, the algorithm immediately switches gears: It outputs the continued [13] b=0 d=-1.39871pt p=-0.234 r=0.088 (dt=0) (0pt)
fraction for $(Ax_j + B)/(Cx_j + D)$, using Euclid's algorithm, and terminates. [14] b=0 d=2.57269pt p=0.431 r=2.573 (dt=0) (0pt)

THESE ARE THE CHOICES FOR THE PARAGRAPH

1	1	0	22301	semiloose	disc
	2	0	256	decent	glue
2	3	1	74326	veryloose	glue
	4	1	32470	decent	glue
	5	2	1103240	decent	penalty
3	6	4	332470	veryloose	penalty
	7	3	334951	semiloose	penalty
	8	4	282614	decent	penalty
	9	4	32591	decent	glue
4	10	6	595239	veryloose	penalty
	11	9	1161220	veryloose	penalty
5	12	10	609880	veryloose	glue
	13	10	605464	decent	glue
	14	10	607840	semitight	glue
	15	11	1171341	decent	glue
	16	11	1198209	semitight	glue
6	17	12	631329	semiloose	glue
	18	13	616620	semiloose	glue
	19	12	619980	decent	glue
	20	13	605753	decent	glue
	21	14	867940	decent	penalty
	22	15	1421462	decent	penalty
	23	15	1182430	semitight	glue
7	24	17	634033	semiloose	glue
	25	18	620416	semiloose	disc
	26	20	627634	semiloose	glue
	27	19	620101	decent	glue
	28	20	605874	decent	glue
	29	21	1970665	decent	penalty
	30	23	2294146	semitight	penalty
8	31	24	637514	semiloose	glue
	32	25	621937	semiloose	glue
	33	27	620201	decent	glue
	34	28	606070	decent	glue
	35	28	624774	semitight	disc
	36	29	1970765	decent	glue
	37	30	2312982	semiloose	glue
	38	30	2304470	decent	glue
	39	30	2304267	decent	glue
9	40	31	1750135	decent	penalty
	41	32	623233	semiloose	math
	42	32	632421	decent	glue
	43	34	621254	semiloose	glue
	44	33	620301	decent	glue
	45	34	608739	decent	disc
	46	35	634874	decent	glue
	47	36	2221206	decent	penalty
	48	38	2304666	decent	glue
	49	39	2317631	semitight	glue
10	50	41	1727333	semiloose	penalty
	51	42	632590	decent	glue
	52	41	635082	semitight	glue
	53	44	870625	decent	penalty
	54	45	1098883	decent	penalty
	55	45	608839	decent	glue
	56	46	637543	decent	disc
	57	46	649583	semitight	disc
	58	47	2221350	decent	glue
	59	48	2307527	decent	disc
	60	48	2327266	decent	glue
	61	48	2315755	semitight	math
	62	49	2318720	semitight	glue
11	63	51	1749990	semiloose	penalty
	64	51	632786	decent	glue
	65	55	879243	semiloose	penalty
	66	55	1098960	decent	penalty
	67	55	608960	decent	glue
	68	56	638027	decent	glue
	69	58	2223950	decent	disc
	70	58	2233031	semitight	glue
	71	62	2333620	semiloose	glue
	72	61	2325855	decent	glue
	73	61	2325899	decent	glue
	74	62	2331320	decent	disc
	75	61	2330136	semitight	disc
12	76	63	1759399	semiloose	glue
	77	63	1750519	semiloose	glue
	78	64	650011	semiloose	glue
	79	64	632886	decent	glue
	80	65	889343	decent	glue
	81	67	630676	semiloose	disc
	82	67	609104	decent	math
	83	68	638148	decent	glue
	84	67	630841	semitight	glue
	85	68	652552	semitight	disc
	86	70	2243431	decent	glue
	87	69	2235039	semitight	glue
	88	70	2243392	decent	glue
	89	72	2328455	decent	disc
	90	73	2328499	decent	disc
	91	75	2339352	semitight	glue
13	92	77	1777419	veryloose	glue
	93	76	1769568	decent	glue
	94	77	1760744	decent	glue
	95	78	662632	decent	disc
	96	79	643847	semiloose	glue
	97	81	638420	semiloose	glue
	98	79	643910	semitight	glue
	99	81	653801	semitight	disc
	100	82	624104	semiloose	disc
	101	82	609204	decent	glue
	102	85	666013	semitight	disc
	103	86	2243531	decent	glue
	104	88	2243561	decent	glue
	105	91	3474956	veryloose	penalty
14	106	93	1791017	semiloose	glue
	107	94	1774844	semiloose	disc
	108	93	1772168	decent	disc
	109	95	695132	semiloose	disc
	110	96	653256	semiloose	glue
	111	97	641445	semiloose	glue
	112	98	647194	semitight	disc
	113	100	637388	semiloose	disc
	114	101	609429	decent	glue

100 82 67 55 45 34 28 20 13 10 6 4 1
101 82 67 55 45 34 28 20 13 10 6 4 1
102 85 68 56 46 35 28 20 13 10 6 4 1

IT IS POSSIBLE TO MAKE A DIFFERENT CHOICE

15. (This procedure maintains four integers (A, B, C, D) with the invari- [1] b=89 d=32.35059pt p=5.414 r=0.966 (dt=90) (0pt)
ant meaning that “our remaining job is to output the continued fraction for [2] b=3 d=9.34784pt p=1.564 r=0.304 (dt=3) (0pt)
$(Ay + B)/(Cy + D)$, where y is the input yet to come.”) Initially set $j \leftarrow$ [3] b=190 d=47.8333pt p=8.005 r=1.240 (dt=191) (0pt)
$k \leftarrow 0$, $(A, B, C, D) \leftarrow (a, b, c, d)$; then input x_j and set $(A, B, C, D) \leftarrow$ [4] b=103 d=49.7424pt p=8.325 r=1.012 (dt=104) (0pt)
$(Ax_j + B, A, Cx_j + D, C)$, $j \leftarrow j + 1$, one or more times until $C + D$ has the [5] b=5 d=17.98949pt p=3.011 r=0.366 (dt=5) (0pt)
same sign as C. (When $j > 1$ and the input has not terminated, we know that [6] b=7 d=-9.79172pt p=-1.639 r=0.405 (dt=7) (0pt)
$1 < y < \infty$; and when $C + D$ has the same sign as C we know therefore that [7] b=1 d=8.76291pt p=1.467 r=0.180 (dt=1) (0pt)
$(Ay + B)/(Cy + D)$ lies between $(A + B)/(C + D)$ and A/C.) Now comes the [8] b=4 d=13.42082pt p=2.246 r=0.337 (dt=4) (0pt)
general step: If no integer lies strictly between $(A + B)/(C + D)$ and A/C, out- [9] b=3 d=-7.34885pt p=-1.230 r=0.298 (dt=3) (0pt)
put $X_k \leftarrow \lfloor A/C \rfloor$, and set $(A, B, C, D) \leftarrow (C, D, A - X_k C, B - X_k D)$, $k \leftarrow k + 1$; [10] b=0 d=-5.07207pt p=-0.849 r=0.147 (dt=0) (0pt)
otherwise input x_j and set $(A, B, C, D) \leftarrow (Ax_j + B, A, Cx_j + D, C)$, $j \leftarrow j + 1$. [11] b=1 d=-7.32619pt p=-1.226 r=0.211 (dt=1) (0pt)
The general step is repeated ad infinitum. However, if at any time the *final* x_j [12] b=2 d=-5.82181pt p=-0.974 r=0.260 (dt=2) (0pt)
is input, the algorithm immediately switches gears: It outputs the continued [13] b=0 d=-1.39871pt p=-0.234 r=0.088 (dt=0) (0pt)
fraction for $(Ax_j + B)/(Cx_j + D)$, using Euclid's algorithm, and terminates. [14] b=0 d=2.57269pt p=0.431 r=2.573 (dt=0) (0pt)

PROFILING CAN REDUCE LINE SPREAD

So the question is: how good an approximation to σ is $\sigma * W\phi$? But the attentive reader will realize that we have already answered this question in the course of proving the sharp Gårding inequality. Indeed, suppose $\phi \in \mathcal{S}$ is even and $\|\phi\|_2 = 1$, and set $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$. Then we have shown (cf. Remark (2.89)) that $\sigma - \sigma * W\phi^a \in S^{m-(\rho-\delta)}_{\rho,\delta}$ whenever $\sigma \in S^m_{\rho,\delta}$ is supported in a set where $\langle\xi\rangle^{\rho+\delta} \approx a$.

No profiling

So the question is: how good an approximation to σ is $\sigma * W\phi$? But the attentive reader will realize that we have already answered this question in the course of proving the sharp Gårding inequality. Indeed, suppose $\phi \in \mathcal{S}$ is even and $\|\phi\|_2 = 1$, and set $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$. Then we have shown (cf. Remark (2.89)) that $\sigma - \sigma * W\phi^a \in S^{m-(\rho-\delta)}_{\rho,\delta}$ whenever $\sigma \in S^m_{\rho,\delta}$ is supported in a set where $\langle\xi\rangle^{\rho+\delta} \approx a$.

LI:0.000

LI:0.000

Profiling

PROFILING CAN REDUCE LINE SPREAD

So the question is: how good an approximation to σ is $\sigma * W\phi$? But the attentive reader will realize that we have already answered this question in the course of proving the sharp Gårding inequality. Indeed, suppose $\phi \in \mathcal{S}$ is even and $\|\phi\|_2 = 1$, and set $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$. Then we have shown (cf. Remark (2.89)) that $\sigma - \sigma * W\phi^a \in S^{m-(\rho-\delta)}_{\rho,\delta}$ whenever $\sigma \in S^m_{\rho,\delta}$ is supported in a set where $\langle\xi\rangle^{\rho+\delta} \approx a$.

No profiling

So the question is: how good an approximation to σ is $\sigma * W\phi$? But the attentive reader will realize that we have already answered this question in the course of proving the sharp Gårding inequality. Indeed, suppose $\phi \in \mathcal{S}$ is even and $\|\phi\|_2 = 1$, and set $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$. Then we have shown (cf. Remark (2.89)) that $\sigma - \sigma * W\phi^a \in S^{m-(\rho-\delta)}_{\rho,\delta}$ whenever $\sigma \in S^m_{\rho,\delta}$ is supported in a set where $\langle\xi\rangle^{\rho+\delta} \approx a$.

Profiling

HAPPY TEXING! (MICKEP@GMAIL.COM)