Extending Opentype math
—
Making decisions

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with Hans Hagen

Illustrations: Sofia Kockum
Why? Many OpenType math fonts exist (Ulrik's talk), but no standard. “It is a mess.”

What? We present some conclusions. The presentation will be visual, not so technical.

Goal? Simple general solutions that work with most fonts, no hacks required.

Who? We aim at ConTeXt users.

Context? This is part of a bigger math project.
PRESENT AND FUTURE TEAM MEMBERS
AVOID OVERLAPPING DUE TO BAD BOUNDING BOXES

(before) \((f)\)  (after) \((f)\)
BOUNDING BOXES–FONTS ARE DIFFERENT

bonum

latin modern

stix two

concrete
BOUNDING BOXES, WHAT DO WE DO?

- Add italic correction to the width.
- Add right bottom kern instead.
- Ensure that the glyph does not stick out to the left.
BONUM USES A LOT OF ITALIC CORRECTION
LUCIDA USES KERNS INSTEAD OF ITALIC CORRECTION
NO CHANGE FOR SUB/SUPERSCRIPTS

\[ f_0^1 \text{ before} \]
\[ f_0^1 \text{ after} \]
ITALIC CORRECTION, WHAT DO WE DO?

- Italic correction transformed to corner kerns.
- Large operators: if nolimits, then italic correction is converted as for other characters.
- Large operators: if limits, then top and bottom anchoring.

Italic correction is not used (relevant) after that.
STAIRCASE KERNS ARE UNRELIABLE

italic V

upright V
STAIRCASE KERNS, WHAT DO WE DO?

- We transform staircase kerns into corner kerns.
- Use the lowest point; assume it works.
ACCENTS ARE SHIFTED TO THE LEFT, NO WIDTH

bonum
cambria
garamond math
kp math
latin modern
libertinus
lucida
stix two
xcharter
ACCENTS DESERVE THEIR SPACE

\[ \vec{p}', \quad \vec{p}' \]
SOME ANCHOR POINTS NEED TO BE CORRECTED

\[ \hat{a}7 \quad \text{before} \quad \hat{a}7 \quad \text{after} \]
SOME ACCENTS CAN BE SCALED

before

after
FLATTENED ACCENTS REDUCE THE RISK OF LINE SPREAD

\( \hat{a} \hat{A} \hat{C} \hat{c} \) stix two

\( \hat{a} \hat{A} \hat{C} \hat{c} \) cambria
WE CAN FAKE FLATTENED ACCENTS

erewhon before

erewhon after
ACCENTS, WHAT DO WE DO?

- Give all accents a proper bounding box.
- Set anchor points in the middle for some alphabets.
- Stretch and shrink accents.
- Emulate flattened accents.
- Provide support for underaccents.
PRIMES ARE DONE DIFFERENT IN DIFFERENT FONTS

lm st sts lucida ssty

erewhon st sst libertinus stsy1
PRIMES, WHAT DO WE DO?

- Tweak in goodie files.
- Primes always go after superscripts.
\[ \sqrt{\frac{1+x}{1-x}} \]
\[ \sqrt{\frac{1+x}{1-x}} \]
\textbf{ANTYKWA TORUŃSKA HAS A BEAUTIFUL SPIRIT}

\texttt{\switchtobodyfont[antykwa]}
\texttt{\startformula}
\texttt{\setmscale{4000}}
\texttt{\sqrt{\frac{1+x}{1-x}}}
\texttt{\stopformula}

\[\sqrt{\frac{1 + x}{1 - x}}\]
NO RULES, REALLY?

- Indeed, no rules.
- Use glyphs instead (if they exist).
MORE ATOM CLASSES ENABLE A MORE FLEXIBLE INTERATOM SPACING

\[ 3a + \sqrt{2} + 3 \int_{0}^{1} f(x) \, dx = 3 \frac{a}{b} \]
SPACING, WHAT DO WE DO?

• Introduce and set up several new atom classes.
• Make some old auto-inserted spaces configurable.
TWEAK FONT PARAMETERS

\[ h^3 + h_2 + h_2^3 + h' \]
\[ h^3 + h_2 + h_2^3 + h' \]
\[ h^3 + h_2 + h_2^3 + h' \]
FONT PARAMETERS, WHAT DO WE DO?

- We use the ones that make sense; many don't.
- We add a few. This extends OpenType math.
- We tweak them in the goodie files.
\[ f(x) \neq f_1(x) \neq f_{x_2}(x) \neq f_{x_2}\left(\frac{1}{x}\right) \]
WHAT IS NEXT?
BREAKING DISPLAYED FORMULAS, PENALTY SETUP NEEDED

Long formula

\[ \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{199} - \frac{1}{200} = \frac{1}{101} + \frac{1}{102} + ... + \frac{1}{200} \]

Automatically broken

\[ \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{199} - \frac{1}{200} = \frac{1}{101} + \frac{1}{102} + ... + \frac{1}{200} \]

Breakpoint by setting a penalty

\[ \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{199} - \frac{1}{200} = \frac{1}{101} + \frac{1}{102} + ... + \frac{1}{200} \]
Three straight lines parallel to the sides of a triangle meet at one point, the sides of the triangle cutting off the line segments of length $x$ each. Find $x$ if the lengths of the triangle's sides are $a$, $b$ and $c$.

1. Nothing special.

2. \texttt{\textbackslash preshortinlinepenalty10000}

3. \texttt{\textbackslash preshortinlinepenalty10000} and multiple passes.
A MULTIPASS SETUP (ASK FOR DETAILS)

\startsetups align:pass:tug
  \pretolerance 100
  \tolerance 200
  \parpasses 2
    \indecentparpassclasses
    \tolerance 300
    \extrahyphenpenalty 50
  \next
    \threshold 0.025pt
    \tolerance 350
    \adjustspacing 3
    \adjustspacingstep 1
    \adjustspacingshrink 20
    \adjustspacingstretch 40
    \emergencystretch \textbf{.25\bodyfontsize}
  \relax
\stopsetups
EXTRA PENALTIES AT THE BEGINNING AND END OF FORMULAS CAN IMPROVE LINE BREAKS

Clearly $1 + 2 = 3$. It is amusing to find that $4 + 5 + 6 = 7 + 8$ and perhaps very surprising that $9 + 10 + 11 + 12 = 13 + 14 + 15$. Check that $16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$, and show that this pattern continues *ad infinitum*.

1

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2

1. Nothing special.

2. Here we used

\setmathpostpenalty\mathbinarycode600
\setmathpostpenalty\mathrelationcode400
\mathforwardpenalties 3 200 100 50
\mathbackwardpenalties 3 200 100 50

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15. (This procedure maintains four integers \((A, B, C, D)\) with the invariant meaning that “our remaining job is to output the continued fraction for \((Ay + B)/(Cy + D)\), where \(y\) is the input yet to come.”) Initially set \(j \leftarrow k \leftarrow 0\) \((A, B, C, D) \leftarrow (a, b, c, d)\); then input \(x_j\) and set \((A, B, C, D) \leftarrow (Ax_j + B, A, Cx_j + D, C)\), \(j \leftarrow j + 1\), one or more times until \(C + D\) has the same sign as \(C\). (When \(j > 1\) and the input has not terminated, we know that \(1 < y < \infty\); and when \(C + D\) has the same sign as \(C\) we know therefore that \((Ay + B)/(Cy + D)\) lies between \((A + B)/(C + D)\) and \(A/C\).) Now comes the general step: If no integer lies strictly between \((A + B)/(C + D)\) and \(A/C\), output \(X_k \leftarrow \lfloor A/C \rfloor\), and set \((A, B, C, D) \leftarrow (C, D, A - X_k C, B - X_k D)\), \(k \leftarrow k + 1\); otherwise input \(x_j\) and set \((A, B, C, D) \leftarrow (Ax_j + B, A, Cx_j + D, C)\), \(j \leftarrow j + 1\). The general step is repeated ad infinitum. However, if at any time the final \(x_j\) is input, the algorithm immediately switches gears: It outputs the continued fraction for \((Ax_j + B)/(Cx_j + D)\), using Euclid’s algorithm, and terminates.
IT IS POSSIBLE TO MAKE A DIFFERENT CHOICE

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So the question is: how good an approximation to $\sigma$ is $\sigma * W\phi$? But the attentive reader will realize that we have already answered this question in the course of proving the sharp Gårding inequality. Indeed, suppose $\phi \in \mathcal{S}$ is even and $\|\phi\|_2 = 1$, and set $\phi^a(x) = a^{n/4} \phi(a^{1/2}x)$. Then we have shown (cf. Remark (2.89)) that $\sigma - \sigma * W\phi^a \in S_{\rho,\delta}^{m-(\rho-\delta)}$ whenever $\sigma \in S_{\rho,\delta}^m$ is supported in a set where $\langle \xi \rangle^{\rho+\delta} \approx a$. 

No profiling

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Profiling
HAPPY $\LaTeX$ING! (MICKEP@GMAIL.COM)

\begin{center}
\includegraphics{image.png}
\end{center}

\texttt{\textbackslash cowprod}