First Problem Assignment

EECS 401

Assigned on: January 13, 2006 Due on: January 20, 2006

PROBLEM 1 (10 points) Fully explain your answers to the following questions.

(a) If events A and B are mutually exclusive and collectively exhaustive, are A^c and B^c mutually exclusive?

> **Solution** $A^c \cap B^c = (A \cup B)^c = S^c = \varnothing$. Thus the events A^c and B^c are mutually exclusive.

(b) If events A and B are mutually exclusive but not collectively exhaustive, are A^c and B^c collectively exhaustive?

> **Solution** Let $C = (A^c \cup B^c)^c$, that is the part that is not contained in $A^c \cup B^c$. Using De Morgan's Law $C = A \cap B = \varnothing$. Thus, there is nothing that is not a part of A^c or B^c. Hence, A^c and B^c are mutually exhaustive.

(c) If events A and B are collectively exhaustive but not mutually exclusive, are A^c and B^c collectively exhaustive?

> **Solution** As in previous part, let $C = (A^c \cup B^c)^c = A \cap B$ which is not null. Thus, A^c and B^c are not mutually exhaustive.

PROBLEM 2 (5 points) Joe is a fool with probability 0.6, a thief with probability 0.7, and neither with probability 0.25.

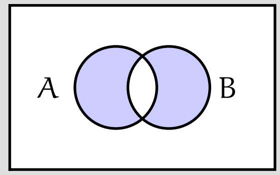
(a) Determine the probability that he is a fool or a thief but not both.

> **Solution** Let A be the event that Joe is a fool and B be the event that Joe is a thief. We are given that
>
> $$\Pr(A) = 0.6$$
> $$\Pr(B) = 0.7$$
> $$\Pr\left((A \cup B)^c\right) = 0.25$$

This implies

$$\Pr\left(A \cup B\right) = 1 - \Pr\left((A \cup B)^c\right) = 0.75$$
$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B) = 0.55$$

The event that he is a fool or a thief but not both is given by $(A \cap B^c) \cup (A^c \cap B)$. Looking at the Venn diagram, the probability should be



$$\Pr\left((A \cap B^c) \cup (A^c \cap B)\right) = \Pr(A) + \Pr(B) - 2\Pr(A \cap B) = 0.2 \quad (1)$$

We can also derive this as follows

$$(A \cap B^c) \cup (A^c \cap B) = (A \cup B) \cap (A \cap B)^c$$

Thus,

$$\begin{aligned}\Pr\left((A \cup B) \cap (A \cap B)^c\right) &= \Pr(A \cup B) + \Pr\left((A \cap B)^c\right) - \Pr\left((A \cup B) \cup (A \cap B)^c\right) \\ &= \Pr(A \cup B) + 1 - \Pr(A \cap B) - \Pr(S) \\ &= \Pr(A \cup B) - \Pr(A \cap B) = 0.2\end{aligned}$$

This is the same expression as (1).

(b) Determine the conditional probability that he is a thief, given that he is not a fool.

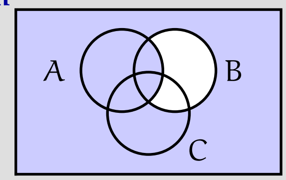
Solution We need to find $\Pr(B \mid A^c)$. We know that

$$\begin{aligned}\Pr(B|A^c) &= \frac{\Pr(B \cap A^c)}{\Pr(A^c)} = \frac{\Pr(B) - \Pr(B \cap A)}{1 - \Pr(A)} \\ &= \frac{0.7 - 0.55}{1 - 0.6} = 0.375\end{aligned}$$

PROBLEM 3 (15 points) Express each of the following events in terms of the events A, B and C as well as the operations of complementation, union and intersection. In each case draw the corresponding Venn diagram.

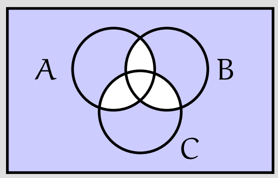
(a) at least one of the events A, B, C occurs;

Solution $A \cup B \cup C$



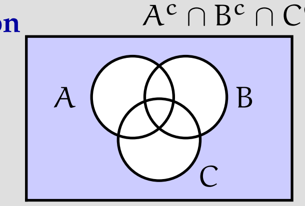
(b) at most one of the events A, B , C occurs;

Solution $[(A \cap B) \cup (B \cap C) \cup (C \cap A)]^c$



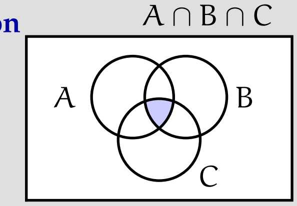
(c) none of the events A, B , C occurs;

Solution $A^c \cap B^c \cap C^c$



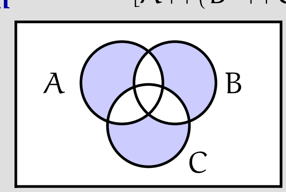
(d) all three events A, B , C occur;

Solution $A \cap B \cap C$



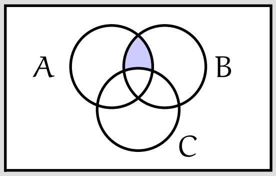
(e) exactly one of the events A, B , C occurs;

Solution $[A \cap (B^c \cap C^c)] \cup [B \cap (C^c \cap A^c)] \cup [C \cap (A^c \cap B^c)]$



(f) events A and B occur, but not C ;

Solution $A \cap B \cap C^c$



(g) either event A occurs or, if not, then B also does not occur.

Solution $A \cup (A^c \cap B^c)$