International Journal for Uncertainty Quantification, x(x): 1–5 (2017)

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Original Manuscript Submitted: mm/dd/yyyy; Final Draft Received: mm/dd/yyyy

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1. INTRODUCTION

Heat is difficult to measure, even at macroscales! At macroscales, key quantities of interest for heat transfer are temperature, heat flux, and thermophysical properties such as thermal conductivity, specific heat, etc. Conductive and convective heat transfer at macroscale are usually governed by diffusion processes because heat carriers (molecules, electrons, phonons, etc.) in these processes have short mean free paths and short wavelengths. In micro- and nanostructures, however, the mean free paths and even wavelengths of heat carriers become comparable or longer than the characteristic length involved in the transport process. Heat transfer can no longer be described by established theories applicable to macroscale. It is precisely these deviations from continuum that have drawn significant interests from scientific communities to understand micro-/nanoscale heat transfer. Such understandings have potential impacts over a wide range of applications, from microelectronics to energy conversion. Experimentally probing heat transfer in micro-/nanostructures is essential for scientific and technological endeavors, and significant progress has been made over the last two decades. This volume aims to provide a summary of the advances made in probing micro-/nanoscale heat transfer.

$$R(E) = \frac{\kappa}{2\eta} \int_{\Omega} (\nabla E \cdot \nabla E + \varepsilon)^{\eta} \, d\Omega \tag{1}$$

In many *gasoline direct injection* (GDI) *engines hollow cone* sprays generated by swirl or outwardly opening injectors are applied. In this study the spray of an outwardly opening injector is investigated. According to the geometrical shape of an outwardly opening pintle nozzle the fuel exits from an annular gap. Previous investigations have already shown that the hollow cone spray leaving this nozzle is composed of many single strings instead of a continuous conical sheet.

2. PROBLEM DEFINITION

In this section, we follow the notation in [1,2]. Define a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with sample space Ω which corresponds to the outcomes of some experiments, $\mathcal{F} \subset 2^{\Omega}$ is the σ-algebra of subsets in Ω (these subsets are called events) and $\mathcal{P} : \mathcal{F} \to [0,1]$ is the probability measure. Also, define D as a d-dimensional bounded domain $D \subset \mathbb{R}^d$ $(d = 1,2,3)$ with boundary ∂D. We are interested to find a stochastic function $u : \Omega \times D \to \mathbb{R}$ such that for $\mathcal{P}$-almost everywhere (a.e.) $\omega \in \Omega$, the following equation holds:

$$\mathcal{L}(\boldsymbol{x}, \omega; u) = f(\boldsymbol{x}, \omega), \ \ \forall \boldsymbol{x} \in D, \tag{2}$$

and

$$\mathcal{B}(\boldsymbol{x}; u) = g(\boldsymbol{x}), \ \ \forall \boldsymbol{x} \in \partial D, \tag{3}$$

where $\boldsymbol{x} = (x_1, \ldots, x_d)$ are the coordinates in $\mathbb{R}^d$, $\mathcal{L}$ is (linear/nonlinear) differential operator, and $\mathcal{B}$ is a boundary operator. In the most general case, the operators $\mathcal{L}$ and $\mathcal{B}$ as well as the driving terms f and g, can be assumed random. We assume that the boundary has sufficient regularity and that f and g are properly defined such that the problem in Eqs. (2)–(3) is well-posed $\mathcal{P}$ -a.e. $\omega \in \Omega$.

2.1 The Finite-Dimensional Noise Assumption and the Karhunen-Loève Expansion the Finite-Dimensional Noise Assumption and the Karhunen-Loève Expansion

Any second-order stochastic process can be represented as a random variable at each spatial and temporal location. Therefore, we require an infinite number of random variables to completely characterize a stochastic process. This poses a numerical challenge in modeling uncertainty in physical quantities that have spatio-temporal variations, hence necessitating the need for a reduced-order representation (i.e., reducing the infinite-dimensional probability space to a finite-dimensional one). Such a procedure, commonly known as a 'finite-dimensional noise assumption' [3,4], can be achieved through any truncated spectral expansion of the stochastic process in the probability space. One such choice is the Karhunen-Loève (K-L) expansion [2].

2.1.1 Suspended Structures

Suspended structures used for nanowire thermal conductivity measurements serve as a good example. From the entire volume, it is clear that significant progress has been made in experimental techniques to probe nanoscale heat transfer phenomena, and the experimental results have led to new understandings of heat transfer physics, generated new challenges, and opened new opportunities. From my own perspective, the following are some significant challenges.

2.1.1.1 Suspended Structures

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Theorem 1. *There exists a unique solution* $u_n \in L^2\left(O, H_0^1(D)\right)$ *to the problem* (2) *and* (3) *for* $n = 0$*, and the problem* (2)–(4) *for* $n \geq 1$*. In addition, if* $f \in L^2\left(O, H^{-1+\sigma}(D)\right)$ *for* $\sigma \in (0,1]$*, it holds that*

$$E(|u_n|^2_{H^{1+\sigma}(D)}) \leq C_0^{n+1}\ E(f^2_{H^{-1+\sigma}(D)}), \tag{4}$$

for some constant C_0 *independent of* n *and* s.

Proof. For $n = 0$, the existence of the weak solutions can be deduced from the Lax-Milgram theorem, and the desired energy estimate,

$$E(|u_0|^2_{H^{1+\sigma}(D)}) \leq \tilde{C}_0\ E(f^2_{H^{-1+\sigma}(D)}),$$

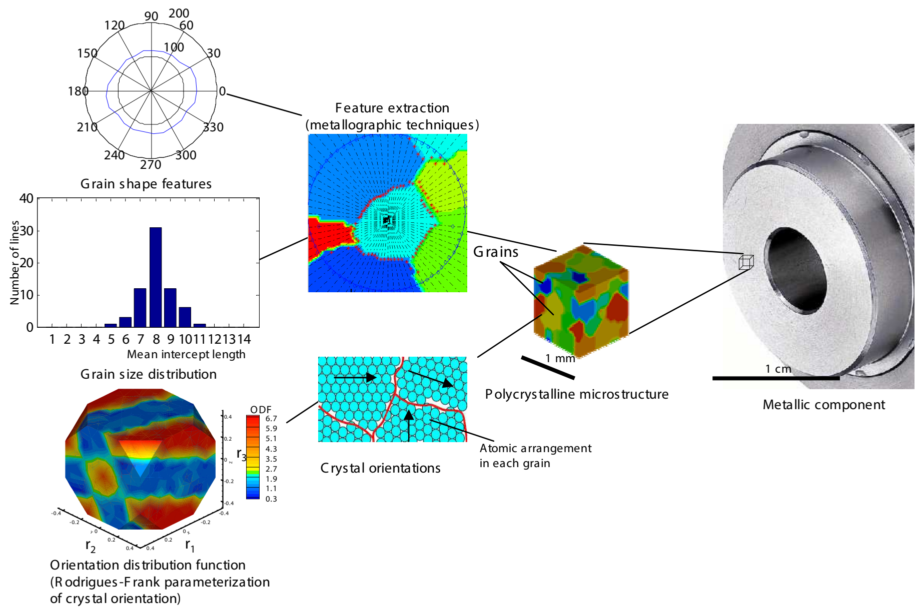
This completes the proof. □

[†] Suspended structures used for nanowire thermal conductivity measurements serve as a good example.

3. ADAPTIVE SPARSE GRID COLLOCATION METHOD (ASGC)

In this section, we briefly review the development of the ASGC strategy. For more details, the interested reader is referred to [1,5].

The basic idea of this method is to have a finite element approximation for the spatial domain and approximate the multi-dimensional stochastic space Γ using interpolating functions on a set of collocation points $\{\mathbf{Y}_i\}_{i=1}^{k} \in \Gamma$. Suppose we can find a finite element approximate solution u to the deterministic solution of the problem in Eq. (2),



(a) (b)

FIG. 1: Visualization setup. (a) Some description for left part. (b) Some description for right part.

TABLE 1: Injected fuel mass

Injection duration Δt **(µs)**	**Injection pressure (MPa)**	**Fuel mass (mg)**
300	1	1.7
300	2	3.0
300	3	4.0
300	4	4.7
300	5	5.4
300	6	6.0
300	7	6.5
300	8	6.9
300	9	7.2
300	10	7.6
300	11	7.7
300	12	8.0
100	10	0.6
200	10	4.3
400	10	10.3[a]

[a] the first note.

Algorithm 1: Block-circulant embedding method (BCEM)

Given $N \in \mathbb{Z}^d, x_{\mathbf{0}} \in \Omega$, and strictly positive valued vector $h \in \mathbf{R}^d$,
Step 1. Choose a vector $m \in \mathbb{Z}^d$ such that $m[i] \geq 2N[i]$ for all $1 \leq i \leq d$.
Step 2. Compute the first block row of the circulant matrix C as described.
Step 3. Compute the block-diagonal matrix $\Lambda = \mathrm{diag}(\Lambda_0, \cdots, \Lambda_{\overline{m}-1})$.

we are then interested in constructing an interpolant of u by using linear combinations of the solutions $u(\cdot, \mathbf{Y}_i)$. The interpolation is constructed by using the so called sparse grid interpolation method based on the Smolyak algorithm. In the context of incorporating adaptivity, we have chosen the collocation point based on the Newton-Cotes formulae using equidistant support nodes. The corresponding basis function is the multi-linear basis function constructed from the tensor product of the corresponding one-dimensional functions.

Any function $f \in \Gamma$ can now be approximated by the following reduced form:

$$f\big(\boldsymbol{x}, \mathbf{Y}\big) = \sum_{\|\mathbf{i}\| \leqslant N+q} \sum_{\mathbf{j}} w^{\mathbf{i}}_{\mathbf{j}}(\boldsymbol{x}) \cdot a^{\mathbf{i}}_{\mathbf{j}}(\mathbf{Y}), \tag{5}$$

where the multi-index $\mathbf{i} = (i_1, \ldots, i_N) \in \mathbb{N}^N$, the multi-index $\mathbf{j} = (j_1, \ldots, j_N) \in \mathbb{N}^N$ and $\|\mathbf{i}\| = i_1 + \cdots + i_N$. q is the sparse grid interpolation level and the summation is over collocation points selected in a hierarchical framework [1,6]. Here, $w^{\mathbf{i}}_{\mathbf{j}}$ is the hierarchical surplus, which is just the difference between the function value at the current point and interpolation value from the coarser grid. The hierarchical surplus is a natural candidate for error control and implementation of adaptivity.

Heat flux cross small structures such as nanowires is very tiny, and high-sensitivity heat flux meters are needed for thermal measurements. Similar to macroscale heat flux measurements, nanoscale heat flux measurements usually require knowing temperature differences between two points and thermal resistance between the same points, from which the heat flux can be calculated. The key for small heat flux measurements is to create structures with large thermal resistance values between the two temperature measurements points.

4. CONCLUSIONS

Significant progress has been made in terms of tailoring the radiative properties with micro-/nanostructured materials. Rapid developments have been made in fabricating periodic gratings and nanostructured periodic arrays of metal materials over thin films and multilayers. Magnetic responses have been demonstrated in the infrared and even visible spectral regions. Further research is needed to understand the coupling mechanisms between various modes and localized surface plasmons. While FDTD and RCWA can be used to calculate the radiative properties for engineered surfaces with micro-/nanostructures, faster computational algorithms are needed with complicated structures to assist the design for specific applications. The optical constants are often different in the nanostructured materials as compared with the bulk solid. Furthermore, for high-temperature applications, the chemical and thermal stability must be considered, as well as the size- and temperature-dependent optical constants of the materials.

1. Aligned metallic nanowires may exhibit unique optical and thermal radiative properties due to the surface-enhanced absorption and scattering, anisotropic dielectric function, and magnetic resonance (between parallel wires), and may be applied to diffraction optics as well as IR polarizers and in the control of optical and radiative properties

2. Detailed models considering both surface scattering and volume scattering will allow better understanding of the radiative transfer in these inhomogeneous structures. Methods for fabricating more uniform structures in large areas with a high yield are still needed.

3. Measurements at longer wavelengths, i.e., mid- to far-IR, will help in understanding the effective medium behavior as well as the magnetic response.

VACNT arrays show great promise for radiometric applications as nearly perfect absorbers and emitters in a broad spectral region. EMT appears to be able to describe the visible and infrared properties of CNT arrays reasonably well. Specular and diffuse black materials made of SWCNTs or MWCNTs should be valuable for space-borne radiometric systems, high-power laser radiometers, absolute cryogenic radiometers, and infrared calibration facilities. The chemical stability and mechanical rigidity of these structures also need to be further investigated. A challenge that exists in the materials growth process is how to control the growth conditions so that arrays with a controllable degree of alignment and surface morphology can be reproduced. However, this error indicator is too sharp and may result in a non-terminating algorithm.

ACKNOWLEDGMENTS

This research was supported by the Computational Mathematics program of AFOSR (grant FA9550-07-1-0139).

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