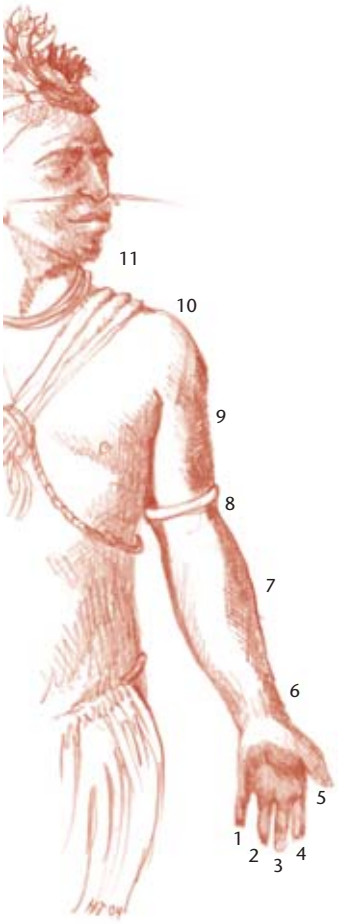
A New Numeral System

by Joel Steinberg



A little research into the history of numeral or counting systems reveals a progression from the very simple to the amazingly complex. We tend to think of simple or "primitive" systems (such as the unary system, where each object to be counted is represented by a hash mark or tally and named not "1, 2, 3," but "1, 1, 1,") as ancient history. But some modern day cultures maintain these systems. They have little need for such abstractions as numerals for groupings of things.

In parts of Papua New Guinea, for instance, the method of counting is still based on the human body. The name for a number, five, for example, might be the same as the name for fingers, because there are five digits on a hand. Or the word used for the upper arm and nine might be the same, because starting with the fingers and counting upward, the upper arm is the ninth point along the way.

In some Kurdish regions people use a system for multiplying on their fingers; in Korea people developed Chisenbop where the fingers and thumbs work as a holding place in the same way an abacus does. (See http://klingon.cs.iupui.edu/~aharris/chis/chis.html for a tutorial in Chisenbop online).

The Incan Quipu (knotted strings) of the Andes region, the Romans' horizontal counting board and the Chinese abacus are examples of ancient tools used to manipulate numbers in a positional system. A positional system is one such as ours where the position of the numeral determines its value. Exploring these different methods of holding numbers, or keeping track of numbers, with students is a wonderful way to open the world of mathematics to different learners. Consider those students who work well with kinesthetic information, or those who need concrete objects to calculate most easily. Providing experiences in different base systems and introducing counting games can help cultivate a rich understanding of place value and operations.

We don't usually think of new number systems being invented in these modern times, but it happens! What follows are excerpts from an article ***Connect*** *received from a medical doctor who developed a system of counting and manipulating numbers. Learning and practicing this system could serve to inspire students to design their own systems and teach them to others.*

—EDITOR

Funforms is a new mathematical numerical notational and learning system. "Fun," because it is *fun*damental, *fun*ctional and *fun*. "Forms" stands for formulae. A colleague of mine, Harold Larson, and I invented and developed it more than twenty years ago.

Funforms are basically binary in design. They operate mechanically, somewhat like an abacus or a slide rule. Using an arithmetic system in counting, numbers progress: one, two, three, etc. Funforms use a geometric progression: one, two, four, eight, sixteen, etc. (In Funforms, the arithmetic progression is preserved, however, if one looks at the exponents.)

Some definitions are required to introduce the system:

- Whole integers begin at **Unity point**. A number value of one is represented by a flag at unity point.
- The **staff** is a vertical line on which numerical values are recorded.
- **Flags** are lines drawn perpendicular to the staff (at available positions).
- There are locations on the staff where flags can be drawn or not drawn.
- When a flag is drawn to the right of

the staff it indicates that the numerical value of that particular number is present in the figure being represented.

- If a flag is drawn to the left of the staff that indicates that the negative value of that particular number is present in the Funform being depicted.
- Flags drawn above unity point have fractional value (2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}, etc.)

How to count using Funforms:

- Counting begins at one, as one might expect. A flag drawn to the right of the staff at unity point stands for "1".
- "2" is written at the position immediately below. That is the position that stands for 2^1.
- By writing both 1 and 2, you have written "3."
- The next position down is 2^2. A flag at that position is "4."
- "5" is written by putting a flag at the position for 4 and 1.

The pattern goes on like that. See Figure 1.

one flag at the next position down. (And conversely, any one flag at a given position is the same as two flags at the preceding position up.)

- Flags to the right of the staff have positive value.
- Flags to the left of the staff have negative value.
- When a circumstance exists in which there is a positive value on one side of the staff at a given position and a negative value at the same position, they cancel one another.

It should not surprise the reader to discover that:

- a flag at unity point denotes an odd number. All odd numbers have a flag there.
- If there is no flag at unity point, it is an even number.

Funforms are most easily written on lined or graph paper. The lines should be labeled 1, 2, 4, 8, 16 to begin writing numbers.

Operations

Now that you know how to count, let's talk about what rules are necessary to use Funforms to add.

- You already know that by writing a flag to the right, at unity point you have written the number one.
- Number values increase in a doubling manner as you go down each potential position on the staff. That is what it means to have geometrically progressive system in this case. (1, 2, 4, 8, 16, etc.)
- In a reciprocal manner, number values are reduced by half as you move up the staff, position by position.
- A very important rule is that only one flag can be at any one position (except temporarily during manipulation). [So at any given position there is either no flag or just one flag, when you finish clearing the operations.]
- Because number values double as you move down the staff, two flags at a given position are the equivalent of

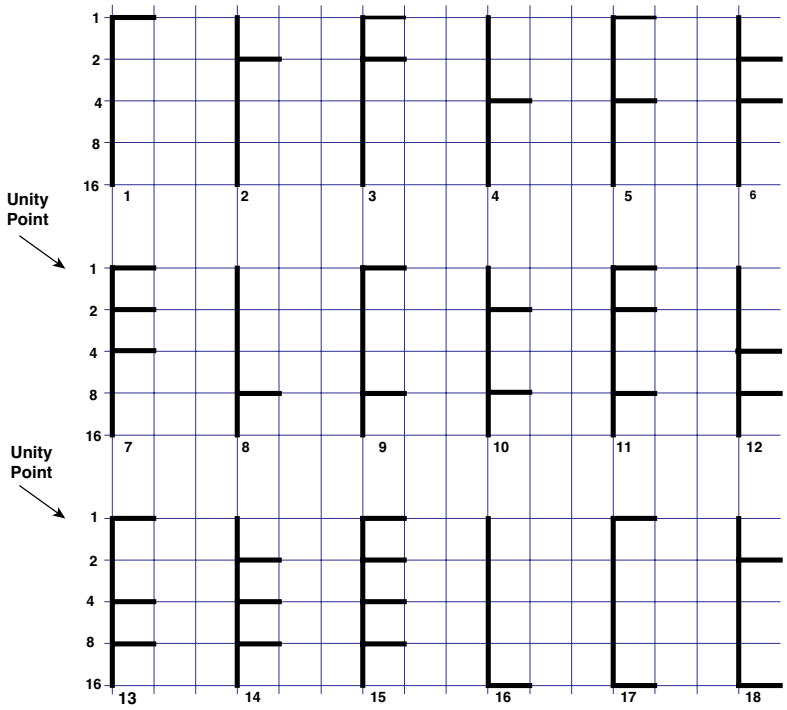


Figure 1

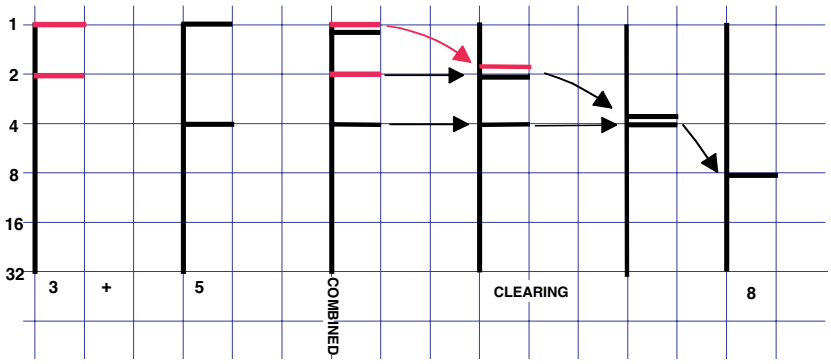


Figure 2

To **add** one simply combines or coalesces all of the number values that are to be added. After combining them, they need to be "cleared." The rule that is used is that any two flags at one position are the equivalent of one flag at the next position down. It is easiest to clear figures beginning at the bottom.

A simple addition problem follows (*See Figure 2*):

I have made the first (3) staff light.

I have left the (5) staff dark.

In the third staff both the flags from 3 *and* the flags from 5 have been combined or coalesced. Note that they appear in exactly the *same positions* that they originally appeared in. The third figure is the result of *combining* the first two figures.

Color should be maintained, so there will be one red and one black flag at the unity postion, one red flag at the two position and one black flag at the 4 position.

Then the task is to clear the excessive flags. Remember that:

1. No more than one flag is allowed at any one position and
2. Two flags at any one position are the same as one flag at the next position down.

The two flags at the unity position become one flag at the next position down (the 2 position). The other flags are simply copied to their proper positions.

Now there are two flags at the 2 position, but that is not allowed in a final figure. So they become one flag at the next position down.

There is already a flag at that position (the 4 position), so again, the two flags at the four position become one flag at the next position down, the 8 position.

The Funforms figure is now in its simplest form and nothing further needs to be or can be done. That is the answer.

Going forward

Possible additional implications of Funforms:

- FunForms will be helping children(and interested adults) understand multibase arithmetic.
- Students of FunForms see the connection between whole numbers and fractions.
- Manipulation of fractions is far more easily done and understood with FunForms than by conventional methods.
- Students of FunForms see that there is no mystery about negative numbers.
- Funforms works like an abacus, but it uses base two and no mechanical device is required.
- Funforms familiarizes students with base 2 and its various permutations, the system used in one form or another, by all digital machines.
- By using a staff with 5 positions beginning at unity point, the student would be dealing with base 16. By using one with 6 positions, the student would be working in base 32. The implications for understanding fundamental aspects of those bases (or others), in the computer field are numerous.
- Funforms could easily be read by an optical scanner and by a human, as contrasted with other artificial constructs like UPC codes.
- Funforms does not use zero as a number or as a placeholder. The absence of a value at any point is made obvious by the absence of any structure indicating a value at that point.

Subtraction, multiplication, division, fractions and working with negative numbers are all possible using this system; to learn how, the balance of this paper is available at no cost by simply requesting it from the author.

Dr. Steinberg is a physician, board-certified in internal medicine and in psychiatry as well as in forensic psychiatry. He has been interested in how symbols are used in cognition for more than 25 years. Dr. Steinberg is happy to communicate by e-mail with any one interested in using Funforms: joxl@sbcglobal.net.