MATH235, Assignment 1

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Contents

1 The Declaration

Plagiarism: Plagiarism involves using the work of another person and presenting it as one's own. For this assignment, the following acts constitute plagiarism:

1. Copying or summarizing another person's work.

2. Where there was collaborative preparatory work, submitting substantially the same final version of any material as another student. Encouraging or assisting another person to commit plagiarism is a form of improper collusion and may attract the same penalties.

1.1 Statement To Be Signed By The Student

1. I have read the definition of plagiarism that appears above.
2. In my assignment I have carefully acknowledged the source of any material which is not my own work.
3. I am aware that the penalties for plagiarism can be very severe.
4. If I have discussed the assignment with another student, I have written the solutions independently.

2 Acknowledgment

All the diagrams in this assignment are drawn by the **PSTricks** package of Herbert Voß.[1]

3 Algebra

3.1 Question One

3.1.1 Part a

$$\begin{cases} T_1 = \frac{20+10+T_2+T_3}{4} \Rightarrow 4T_1 = T_2 + T_3 + 30 \Rightarrow -T_2 + 4T_1 - T_3 = 30 \\ T_2 = \frac{30+20+T_1+T_4}{4} \Rightarrow 4T_2 = T_1 + T_4 + 50 \Rightarrow -T_1 + 4T_2 - T_4 = 50 \\ T_3 = \frac{40+10+T_1+T_4}{4} \Rightarrow 4T_3 = 50 + T_1 + T_4 \Rightarrow -T_1 + 4T_3 - T_4 = 50 \\ T_4 = \frac{40+30+T_2+T_3}{4} \Rightarrow 4T_4 = 70 + T_2 + T_3 \Rightarrow -T_2 - T_3 + 4T_4 = 70 \end{cases}$$

We use 4 variables. We obtain 4 equations. Yes, they are linear.

[1] http://tug.org/PSTricks/main.cgi/

3.1.2 Part b

$$\begin{bmatrix} 4 & -1 & -1 & 0 & 30 \\ -1 & 4 & 0 & -1 & 50 \\ -1 & 0 & 4 & -1 & 50 \\ 0 & -1 & -1 & 4 & 70 \end{bmatrix} \equiv \begin{bmatrix} 4 & -1 & -1 & 0 & 30 \\ 0 & 15 & -1 & -4 & 230 \\ 0 & -1 & 15 & -4 & 230 \\ 0 & -1 & -1 & 4 & 70 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 4 & -1 & -1 & 0 & 30 \\ 0 & 15 & -1 & -4 & 230 \\ 0 & 0 & 224 & -64 & 3680 \\ 0 & 0 & -16 & 56 & 1280 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 4 & -1 & -1 & 0 & 30 \\ 0 & 15 & -1 & -4 & 230 \\ 0 & 0 & 224 & -64 & 3680 \\ 0 & 0 & 0 & 720 & 21600 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 4 & -1 & -1 & 0 & 30 \\ 0 & 15 & -1 & -4 & 230 \\ 0 & 0 & 7 & -2 & 115 \\ 0 & 0 & 0 & 1 & 30 \end{bmatrix}$$

$$\begin{cases} 4T_1 - T_2 - T_3 = 30 \Rightarrow T_1 = 20 \\ 15T_2 - T_3 - 4T_4 = 230 \Rightarrow T_2 = 25 \\ 7T_3 - 2T_4 = 115 \Rightarrow T_3 = 25 \\ T_4 = 30 \end{cases}$$

3.2 Question Two

3.2.1 Part a

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & -2 & -1 \\ 0 & 0 & 4 & 1 \end{bmatrix}$$

3.2.2 Part b

$$\begin{bmatrix}1 & 1 & 2 & 2\\ 1 & 1 & -2 & -1\\ 0 & 0 & 4 & 1\end{bmatrix} \equiv \begin{bmatrix}1 & 1 & 2 & 2\\ 0 & 0 & 4 & 3\\ 0 & 0 & 4 & 1\end{bmatrix}$$
$$\equiv \begin{bmatrix}-2 & -2 & 0 & -1\\ 0 & 0 & 4 & 3\\ 0 & 0 & 0 & 2\end{bmatrix}$$
$$\equiv \begin{bmatrix}-2 & -2 & 0 & -1\\ 0 & 0 & 4 & 3\\ 0 & 0 & 0 & 1\end{bmatrix}$$
$$\equiv \begin{bmatrix}-2 & -2 & 0 & 0\\ 0 & 0 & 4 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$
$$\equiv \begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

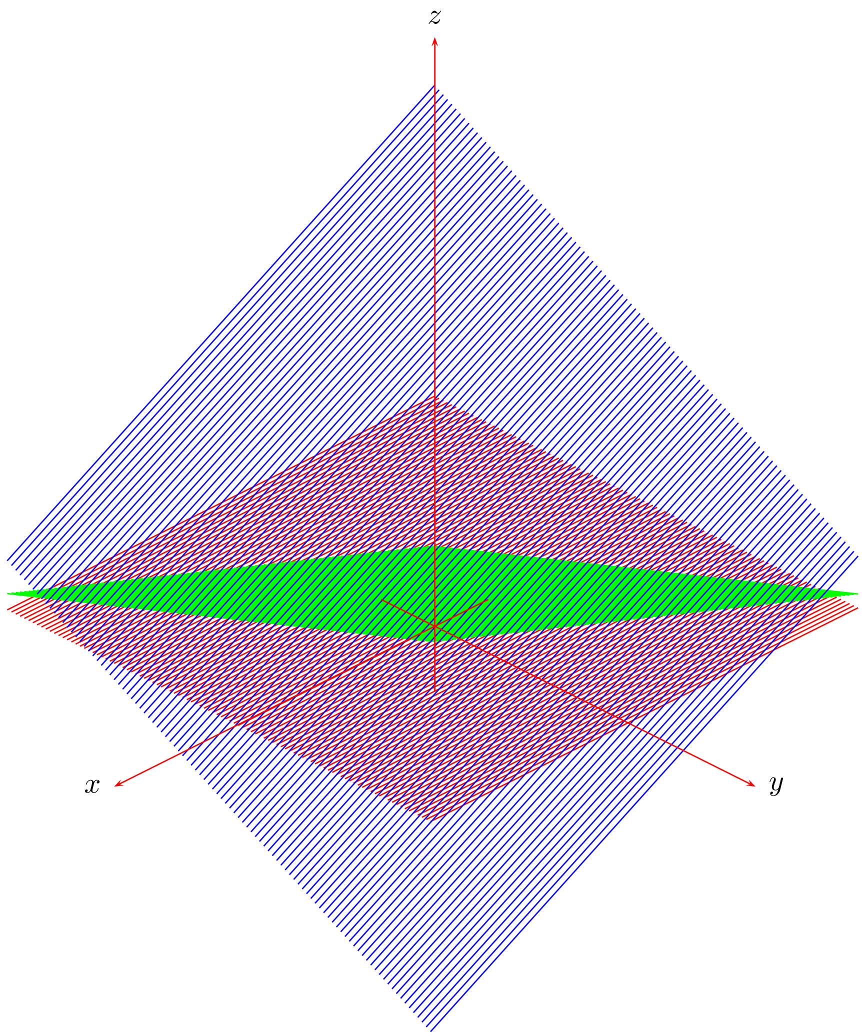
3.2.3 Part c

There is no solution.

3.2.4 Part d

It seems that these three planes do not have a point of intersection common in all of them (some of these planes are parallel). If we solve $x + y + 2z = 2$ and $x + y - 2z = -1$, we see that these two planes are parallel because we get no solution, if we solve $x + y + 2z = 2$ and $4z = 1$ we see that these two planes intersect at all points on the line $y = \frac{3}{2} - x$ and if we solve $x+y-2z = -1$ and $4z = 1$ we see that these two planes intersect at all points on $y = -\frac{1}{2} - x$.

In The diagram blow, the red color shows the graph of $4z = 1$, the green color shows the graph of $x + y - 2z = -1$ and the blue color shows the graph of $x + y + 2z = 2$.



4 Calculus

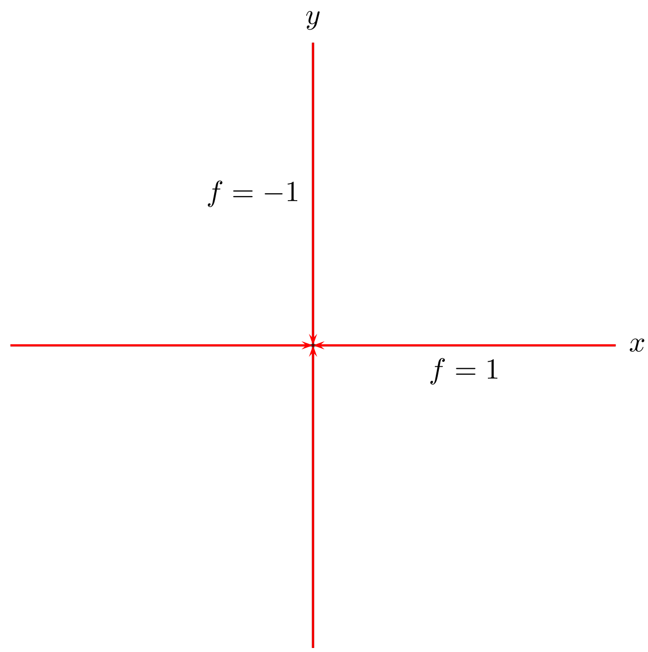
4.1 Question One

Let's first find $\lim_{\mathbf{x}\to\mathbf{0}} f(\mathbf{x})$. First let's approach $\mathbf{0}$ along the x-axis.

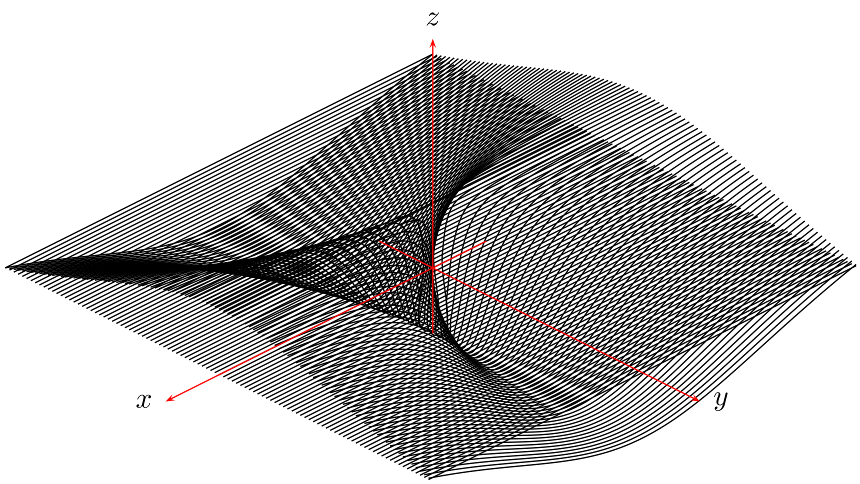
$$\lim_{\mathbf{x}\to\mathbf{0}} f(\mathbf{x}) = \frac{x^2}{x^2} = 1$$

We now approach $\mathbf{0}$ along the y-axis.

$$\lim_{\mathbf{x}\to\mathbf{0}} f(\mathbf{x}) = -\frac{y^2}{y^2} = -1$$



Since f has two different limits along two different lines, the given limit does not exist and therefore the function is not continuous at $\mathbf{0}$. Let's see what does the actual function look like.



4.2 Question Two

$$f\left(\phi\left(t\right)\right)=\frac{2t^2\times at}{t^4+a^2t^2}=\frac{2at^3}{t^2\left(t^2+a^2\right)}=\frac{2at}{t^2+a^2},\qquad\text{and}$$
$$f\left(\psi\left(t\right)\right)=\frac{2t^2\times t^2}{t^4+t^4}=\frac{2t^4}{2t^4}=1$$

4.2.1 Part a

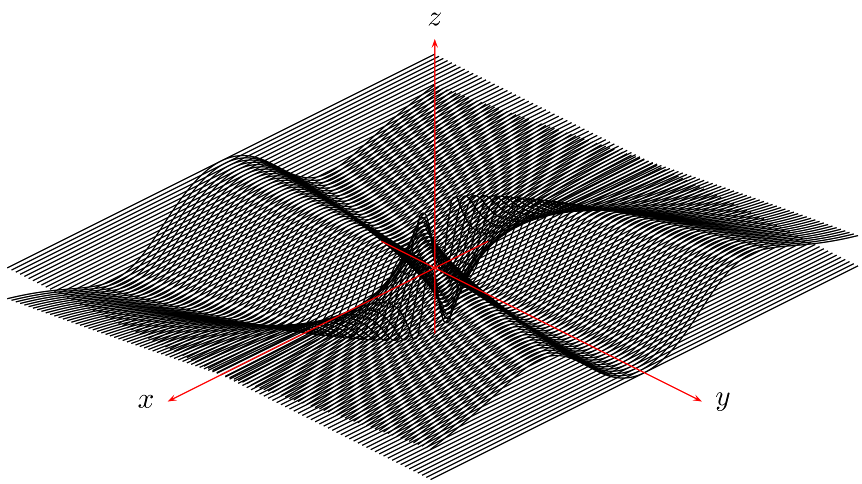
$$\lim_{t\to0}f\left(\phi\left(t\right)\right)=\lim_{t\to0}\frac{2at}{t^2+a^2}=\frac{0}{a^2}=0$$

Since $\lim_{t\to0}f\left(\phi\left(t\right)\right)=f\left(\phi\left(0\right)\right)=0$, then f is continuous on any straight line through origin.

4.2.2 Part b

$$\lim_{t\to 0} f\left(\psi\left(t\right)\right) = \lim_{t\to 0} 1 = 1$$

Since $\lim_{t\to 0} f\left(\psi\left(t\right)\right) \neq f\left(\psi\left(0\right)\right) = 0$, then f is not continuous at origin.



4.3 Question Three

Let $\epsilon > 0$ be given. We want to find a number N such that

$$\|f(\mathbf{a_k}) - f(\mathbf{a})\| < \epsilon \qquad \text{whenever} \qquad k > N$$

since f is continuous at $\mathbf{a}$, we have

$$\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) = f(\mathbf{a})$$

and so there exists $\delta_1 > 0$ such that

$$\|f(\mathbf{x}) - f(\mathbf{a})\| < \epsilon \qquad \text{whenever} \qquad 0 < \|\mathbf{x} - \mathbf{a}\| < \delta_1$$

since $\lim_{k\to\infty} \mathbf{a_k} = \mathbf{a}$, there exists N such that

$$\|\mathbf{a_k} - \mathbf{a}\| < \delta_1 \qquad \text{whenever} \qquad k > N$$

combining these two statements, we see that whenever $k > N$, we have $\|\mathbf{a_k} - \mathbf{a}\| < \delta_1$, which implies that $\|f(\mathbf{a_k}) - f(\mathbf{a})\| < \epsilon$. Therefore, we have proved that $\lim_{k\to\infty} f(\mathbf{a_k}) = f(\mathbf{a})$.