

$$\begin{aligned}\text{Volume} &= \iiint_S \rho^2 \sin \theta \, d\rho \, d\theta \, d\phi \\ &= \int_0^{2\pi} d\phi \int_0^\pi \sin \theta \, d\theta \int_0^R \rho^2 \, d\rho \\ &= \phi \Big|_0^{2\pi} (-\cos \theta) \Big|_0^\pi \frac{1}{3} \rho^3 \Big|_0^R \\ &= 2\pi \times 2 \times \frac{1}{3} R^3 \\ &= \frac{4}{3} \pi R^3\end{aligned}$$