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CŒUR d'ANALYSE

Lectures on Convergence and Continuity
with Discussions and Digressions

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Part I

The Real Numbers

Lecture 1

More or Less: The Algebra and Geometry of Inequalities

1.1 The Number Line

"Numbers " come in several flavors, and in varying degrees of abstraction.

The **natural numbers** $1, 2, 3, \ldots$ are associated with counting–either counting how many elements belong to a set ("cardinal" numbers), or locating a position in a list ("ordinal" numbers). Addition of natural numbers is associated to counting the number of elements in the union $A \cup B$[1] of two disjoint[2] sets A and B: if A has m elements and B has n then $A \cup B$ has $m + n$. If we think of the natural numbers as a list, with the successor $m + 1$ of $m \in \mathbb{N}$ to the right of m, then adding n to m has the effect of moving n places to the right. Multiplying m by n is the same as adding together n copies of m. Addition and multiplication are related by the **distributive law**: $m(n + k) = mk + nk$. The collection of all natural numbers is denoted $\mathbb{N}$.

The **integers** $\mathbb{Z}$ consist of the natural numbers, or in this context the **positive integers** together with the number zero (0) and the **negative integers** $-1, -2, -3, \ldots,$ arranged in succession to the left of $\mathbb{N}$. The identities $n + 0 = n$, $-n + n = 0$ and $-n = (-1)n$ together with the distributive law lead to a unique extension of addition and multiplication from $\mathbb{N}$ to $\mathbb{Z}$. **Subtraction** of n from m is defined as $m - n = m + (-n)$.

[1] The union $A \cup B$ consists of all elements that belong to either A or B or both.

[2] A and B are **disjoint** if they share no common elements–that is, their **intersection** contains no elements–it is "empty": $A \cap B = \emptyset$.