Interview with Alan J. Cain

Jim Hefferon

Alan J. Cain is the author of *Form & Number: A History of Mathematical Beauty* [2]. Details of the technicalities of writing this book are the subject of two recent *TUGboat* articles [3, 4]. (We shall abbreviate the title as *F&N*.)

Jim Hefferon, interviewer: Let's start with your personal history apart from TeX and friends.

Alan J. Cain: I am originally from Scotland. I did my first degree (in mathematics) at the University of Glasgow (1998–2002), then a Ph.D. (again in mathematics) at the University of St Andrews (2002–05). After a three-year post-doctoral fellowship at St Andrews, I moved to Portugal. I worked at the University of Lisbon for a year, then at the University of Porto for five years, after which I was at NOVA University of Lisbon (which, to be clear, is a different institution) until late 2023. As a mathematician, my research started in semigroup theory but later expanded into combinatorics and theoretical computer science.

JH: When and how did you first get interested in TeX and friends?

AJC: I was vaguely aware of LaTeX during my first two years as an undergraduate at the University of Glasgow. At the start of the third year, I attended a short course on LaTeX aimed at students specializing in mathematics or related areas. That was my first experience in actually using it, and I prepared various LaTeX documents during the remainder of my undergraduate studies. I certainly appreciated the high-quality output, although I did little in the way of experimentation.

When I went to St Andrews, I naturally used LaTeX for writing articles. During the first year of my Ph.D., I started to learn MetaPost, largely because I tended to use a lot of diagrams in my work and I wanted them to look good.

I think it was during the second year of my Ph.D. that I developed a desire to have my thesis be typographically distinct. There was no official university LaTeX class. (That may have changed. At the time, the thesis regulations seemed to assume that the manual typewriter was the cutting-edge of technology.) I no longer recall the precise sequence of events, but I certainly bought and read Knuth's *The TeXbook* [9] and I borrowed his collection of papers *Digital Typography* [11] from a colleague, and I must have read through parts of *The LaTeX Companion* [14]. I know I also read Bringhurst's *Elements of Typographic Style* [1] during that year.

The style I developed for my thesis was my first foray into using LaTeX beyond a 'pure user' level. The style comprised a collection of hacks into the standard `book` class, an extension of the `harvard` citation package, and a collection of macros for typesetting epigraphs and making minor typographical tweaks.

I'm not sure when I began using Unicode-aware engines. I must have experimented with XeLaTeX around 2010–11, and I used it to prepare a set of lecture notes on semigroups in 2012. I was certainly using LuaLaTeX by 2017. I think I started using Asymptote around the same time.

JH: Your book *Form & Number* will appeal to mathematicians, to historians of mathematics and science, and to philosophers. The readers of *TUGboat* are a varied group, although an attraction to beauty in technical work is a common thread. But everyone is so busy that perhaps an interested party is unaware of the book's existence. Can you summarize the contents?

AJC: *Form & Number* is an attempt at a comprehensive (within certain bounds) study of the history of mathematical beauty. The aim is to offer an account of the historical development of the experience of beauty in doing or contemplating mathematics, and of the scrutiny to which such experience has been subjected by mathematicians and philosophers. At various points, the relevance of mathematical beauty to works of architects, artists, calligraphers, historians, poets, and theologians also appears. With brief exceptions, the scope of *F&N* is limited to the western tradition of mathematics and philosophy. (At present, I do not consider myself remotely familiar enough with the general intellectual history of (for example) China, India, Japan, ... to even consider including them. Perhaps one day ...)

After an introductory chapter, *F&N* is divided into four parts. The first covers subjects that are almost entirely historical, and so the chapters are arranged more-or-less chronologically from the beginnings of mathematics to the early twentieth century. The chapters in the second part have a thematic division, and deal with matters that are still 'current': for example, beauty as a motivation for mathematics; whether beauty can serve as a guide in mathematics; and the ongoing philosophical debate over whether mathematical beauty is truly aesthetic, only quasi-aesthetic, or simply a name for something else entirely. The third part deals with the role of mathematical beauty in the natural sciences. (It is not an account of the role of (general) beauty in the sciences, which would be another book.) The short

example.org/10.47397/tb/46-2/tb143cain-hefferon

fourth part contains some reflections and notes on some questions that arise.

JH: This is a scholarly work. A key aspect of scholarship is taking pains with the material. I was struck by your work in tracking down exactly what was said by exactly who, and when they said it. Have you spent a great deal of effort on travel to various sites in conjunction with that? Have you had the opportunity to present and discuss the material with other scholars as you developed it? To teach on the subject?

AJC: I did not do any travelling *for F&N*, although whenever I visited another university for a research visit or conference, I always had a 'shopping list' of articles, ebooks, etc. that I wanted, and I would check whether there was institutional access that I didn't have at my home university. And of course I did a certain amount of reading in libraries of universities that I happened to visit.

My work on *F&N* was a hobby and unsupported by grants or other funding, so any travelling for the purpose of research on the book would have been at my own expense. That said, over the years I spent a fair amount of my own money on sources that I wanted to read.

But many of the historical sources that I needed to consult were out of copyright and freely available on the Internet Archive, Gallica, `e-rara.ch`, etc. There was a certain satisfaction in finding and assembling a collection of good-quality scanned volumes of the collected works of Fermat, Descartes, Lagrange, etc. And some editions that are not yet out of copyright are freely available anyway. For example, Kepler's *Gesammelte Werke*,[1] the last volume of which appeared in 2017, has been made available under a Creative Commons Attribution Licence by the Bavarian Academy of Sciences and Humanities, under whose auspices it was edited.

Except for a few friends, no one knew I was working on *F&N* until quite late. I have given a couple of seminars based on certain sections of the book. The audiences were mainly (perhaps entirely) mathematicians. While the reception was generally positive, I have little idea so far what professional historians or philosophers of mathematics think of the work of a dilettante like me!

JH: In a Calculus class, I have just done $e^{i\pi}+1=0$. As with prior times, there are some people who light up but they are by no means everyone, or even half of the class. This is despite my pointing out that something special is happening. These are people who have some ability with technical material since they are in this class. Is there a sense in which appreciation of an element of beauty in Mathematics at all is only something that a relatively few people have?

Following up on that, I enjoyed the book's discussion of fractals. Over the years I have been to many talks about the topic and read a number of books. But I confess that I have never felt an appeal.

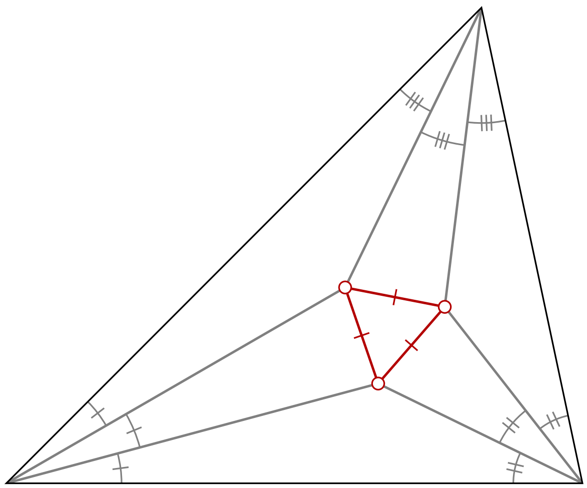
Are there some things that most observers would accept as part of the beauty in the subject and then some are more variable?

AJC: First, the question of what can be loci of mathematical beauty is subject to some dispute. Roughly, theorems, proofs, theories, examples are broadly accepted as potential loci of beauty, although some people question whether a result can be aesthetically judged apart from its proof or from its context within a theory. Definitions, problems, conjectures are often accepted as loci, but, to the best of my knowledge, aesthetic evaluations of them have been subject to little philosophical scrutiny. Mathematical objects have historically often been thought beautiful, but nowadays seem to be less commonly judged aesthetically. (Perhaps this is because it raises questions about the ontology of mathematical objects?) Some people even suggest that mathematical symbolism, separate from its semantic content, can be beautiful. Does this count as *mathematical* beauty?

Diagrams and images like fractals and tilings are perhaps the most interesting borderline cases. Certainly *some* find aesthetic value in them. But is the aesthetic response to the visual stimulus connected to the abstract mathematics? The testimonies of working mathematicians, with some exceptions, suggest that there is such a relationship between the visual beauty of fractals and the abstract beauty of the underlying mathematics. But, frustratingly, the testimonies do not reflect on or explicate this relationship.

Regarding the varying appreciation of beauty in mathematics, I suspect that there is a range of responses rather than a sharp division into those who do appreciate it and those who do not. And there is a similar range of ability to *access* mathematical beauty. If a person does not have a sufficient level of mathematics education, certain kinds of mathematical beauty are simply not accessible to them. Someone with no formal mathematical education can still access and make aesthetic judgements about images such as fractals, Islamic geometrical art, or kolam, which are *somehow* mathematical. A little more education allows one to aesthetically evaluate

[1] `kepler.badw.de/kepler-digital.html`



Morley's trisector theorem: In any triangle, the points of intersection of the trisectors of adjacent angles are the vertices of an equilateral triangle.

elementary mathematics. Degree- or postgraduate-level training grants the access necessary to judge still more mathematics.

There are also some people who accept that there is beauty in higher mathematics, but who lack the ability to access it. Many years ago, I talked to a Japanese artist who, I believe, had no mathematics training beyond high school, but who recognized that mathematics could be an aesthetic pursuit. (I do not recall what examples of mathematical beauty I tried to give during our conversation.) The philosopher Simone Weil is an interesting example. She lamented her inability to perceive the kinds of beauty accessible to her brother André, but the idea of mathematical beauty can be traced through her Christian platonist philosophy.

A person with the requisite level of training to understand a mathematical artefact — a fractal image; a formula; a theorem and proof and the relevant mathematical context — does not necessarily have the same aesthetic response as their peers. In the literature, there are many claims (and some dissents) that mathematicians generally agree on aesthetic judgements, and a few studies that shed some light on the question.

Here is a notable example: Gian-Carlo Rota thought that there would be a consensus on aesthetic value in mathematics. But one of his examples of a non-beautiful result was Morley's trisector theorem (see figure), which many other mathematicians have praised for its beauty.

The specific case of Euler's equation $e^{i\pi} + 1 = 0$ illustrates the question of whether there is agreement on beauty. It is true that this equation is praised more than any other in terms of beauty, elegance, etc. And in surveys and questionnaires of mathematicians, it has been consistently ranked as the most beautiful. But there are complications. Some of the questionnaires asked about Euler's equation in the form $e^{i\pi} = -1$, which some authors have explicitly held to be less beautiful than the 'standard' form, which contains the constants 0, 1, e, π, and i. And those who think the circle constant should be $\tau = 2\pi$ prefer $e^{i\tau} = 1$. One respondent to a questionnaire rated it as very beautiful only as a symbol for the whole of complex analysis.

And there is perhaps a historical mystery. The first known publication with a explicit statement of Euler's equation (in the form $e^{i\pi} = -1$) seems to be an 1813–14 paper by Jacques François on the representation of complex numbers in the plane. (Euler himself only published the formula $e^{i\theta} = \cos\theta + i\sin\theta$.) But I have been unable to find *any* aesthetic judgement of Euler's equation before essays by the chemist and writer François Le Lionnais in 1946 and 1948, who described the equivalent (but surely less beautiful) form

$$\sqrt{-1}^{\sqrt{-1}} = e^{-\pi/2}$$

as having *once* been the most beautiful formula in mathematics; its beauty, he thought, had faded through familiarity. Then Edward Kasner and James Newman described $e^{i\pi}+1 = 0$ as elegant in their 1949 book *Mathematics and the Imagination* [8]. And these are the *only* explicit instances of aesthetic praise of Euler's equation that I have found before 1988, after which it is much more frequently described as beautiful or elegant. The earlier texts I have located that include a judgement of the equation only call it 'remarkable' or 'mysterious' or 'paradoxical'.

Euler's equation should be at least *surprising* when one first learns of it. The number π ultimately derives from (ancient) geometry, i from (Renaissance) algebra, e from (early modern) analysis and yet here they all combine. For some, this surprise is a part of its beauty. But some professional mathematicians hesitate about calling it beautiful: perhaps the equation and the concepts it uses are so embedded in complex analysis that, as Le Lionnais suggested, for some people it becomes so familiar as to breed ennui.

Appreciation of beauty in mathematics is, in one sense, highly culturally dependent, for mathematical education is a very particular form of cultural influence. And I wonder if mathematical training actually inclines people to declare Euler's equation (and perhaps other results) beautiful, *even if* they personally have no aesthetic response. It's like viewing a great masterpiece of art: regardless of one's own feelings,

one does not want to be thought a philistine, and it's the 'done thing' to express appreciation.

Finally, there seem to be at least some professional mathematicians over whom beauty has no hold. One respondent interviewed for a study on mathematical practice said: 'Beauty doesn't matter. I have never seen a beautiful mathematical paper in my life.' Personally, I feel a twinge of pity for someone who pursues mathematics and yet remains immune to its beauty.

JH: I taught at a liberal arts college for many years. We had a course where senior students presented on a math topic. Every year someone presented on the Golden Ratio, and I groaned every year. Finally I outlawed it and although it was a struggle, I was happier in the subsequent classes. I felt I had struck a blow.

Anyway, I was delighted to read your comments on the topic. If for nothing else, thank you for that.

AJC: When I first thought about writing on mathematical beauty, I also thought that the golden ratio would be an important theme. And, in a sense, it *is* important, if only because there has been so much confusion on the topic. I remember reading early some of the more sceptical literature on the subject, like George Markowsky's paper *Misconceptions about the Golden Ratio* [12], which helped me clarify my views. And Roger Herz-Fischler's book *A Mathematical History of the Golden Number* [7] is an excellent scholarly guide to its purely mathematical history.

As I said in *F&N*, there seems to be precisely *nothing* in the literature about using the extreme and mean ratio — I now prefer to avoid the emotive term 'golden' — in architecture or visual art until the second edition of Jean-Étienne Montucla's (1725–99) *Histoire des Mathématiques* [15] where Montucla made the demonstrably incorrect claim that Luca Pacioli (c1447–1517) had published illustrations of its application to architecture. As a historical coincidence, this was not long after the term 'golden ratio' was coined by Samuel Traugott Gehler (1751–95). In the second half of the nineteenth century, Adolph Zeising (1810–76) adopted the extreme and mean ratio as the foundation of his aesthetic system and tried to find instances of it in art, and Gustav Fechner (1801–87) made a much-misreported experiment that he thought showed the visual aesthetic superiority of the extreme and mean ratio.

The actual confusion of the mathematical and (supposed) visual beauty of the extreme and mean ratio seems to have begun in the early twentieth century and gradually became embedded in the literature. I hope that at least I managed to disentangle them.

JH: Are there some principles in what leads to a thing in Mathematics being considered beautiful? To me, Kleene's fixed-point theorem in the Theory of Computation certainly qualifies. It is brief, relatively non-technical, applies to a very wide class of functions, and has practical ramifications such as the existence of quines, and of reflection in general. But I imagine many others would not find it to be a gem (for one thing, the traditional proof is often considered mysterious). Is there some guidance you can give toward picking out results that are likely to be a hit, say in a talk for prospective students?

AJC: I think the starting point for any discussion of criteria for beauty has to be what I call the Hardian triad: unexpectedness, inevitability, and economy, which G.H. Hardy considered to be purely aesthetic qualities possessed by beautiful theorems and proofs.

Some authors add seriousness, generality, and depth to make 'Hardy's six', but I think that Hardy did not consider these to be aesthetic qualities: for instance, he thought that a chess problem can be beautiful but is never serious. You mentioned the wide range of applicability and consequences of the fixed-point theorem as a reason to find it beautiful. These certainly contribute to the theorem being serious, general, and (I think) deep. But do they contribute to making it beautiful? Perhaps these qualities rather *enhance* beauty than help ground it.

Returning to the Hardian triad: I think the unexpectedness must be of a 'good' kind: the revelation of some previously unseen pattern that one can see in retrospect; it cannot be a 'deus ex machina' step that serves as a black box. This 'good' unexpectedness is compatible with inevitability.

Inevitability is perhaps related to explanatoriness, and I think that there is some correlation between a proof being explanatory and being beautiful. Precisely what constitutes explanation in mathematics is in itself a complicated question, and there are arguments about whether a proof by induction can be explanatory. But it is easy to give examples of proofs that explain and proofs that 'merely demonstrate' (there are several examples in the chapter 'The Smile of Reason').

I emphasize *some* correlation. I do not think there is an implication in either direction. In particular, there is a conflict regarding *reductio ad absurdum* proofs, which many think can be beautiful (including of course Hardy, who thought it 'one of the mathematician's finest weapons') but which many also think non-explanatory. (An early modern example of this position is found in Antoine Arnauld and Pierre

Nicole's *Port-Royal Logic*, first published in 1662:[2] 'our mind is not satisfied unless it knows not only that something is, but why it is, which is not learned from a proof that reduces to absurdity.')

Economy certainly contributes to beauty, but (unlike some authors) I think that economy is distinct from simplicity. For a proof to be economical, it must not rely on 'heavy machinery' (meaning techniques and concepts much more difficult than the theorem to be proved). For a proof to be simple, on the other hand, it cannot be too 'wide' (in the sense of requiring much 'working information' to be kept in mind or otherwise referred to). I think that a proof can be economical but not simple (elementary concepts are used, but much information has to be kept at hand), or simple but not economical (heavy machinery is used, but all the complexity is buried within the advanced concepts). I do not claim that these are complete definitions or characterizations of economy and simplicity, but I hope they make the difference clear.

Of course, our (the mathematical community's) aesthetic criteria may vary over time. But perhaps there is some underlying pattern. For example, there is a notion of 'aesthetic induction', formulated as part of a rationalist account of beauty in the sciences by James W. McAllister in his book *Beauty and Revolution in Science* [13]. McAllister argued that scientists' aesthetic criteria are shaped by previous empirical success and thus allow aesthetic value to serve as a guide. At some points, empirical success and existing aesthetic criteria come into conflict and the resulting rupture — a revolution — leads to the adoption of new (possibly radically different) criteria.

McAllister and another author, Ulianov Montano, have considered how to apply this notion of aesthetic induction to mathematics. If they are right, new developments in mathematics will change the criteria that influence our judgements of mathematical beauty. McAllister suggested that such a shift would occur with regard to computer proofs, which have often been aesthetically disparaged, as they became more successful. While computer proofs have become *accepted*, and nowadays there tends to be little doubt among mathematicians that they genuinely establish results, I see little evidence that they are coming to be seen as beautiful. (And the lack of explanatoriness of computer proofs might also contribute to them not being seen as beautiful.)

JH: Do you know of any work done on beauty in nearby areas, such as algorithms or programs?

[2] See `plato.stanford.edu/entries/port-royal-logic/`.

AJC: I'm afraid I can only offer some isolated comments here.

There is certainly recognition of beauty and elegance in algorithmics and programming, but to the best of my knowledge there has been little exploration or philosophical scrutiny of it. In the first chapter of *The Art of Computer Programming*, Donald Knuth pointed to aesthetic judgement of algorithms:

> we want algorithms that are *good* in some loosely defined aesthetic sense. One criterion of goodness is the length of time taken to perform the algorithm [...]. Other criteria are the adaptability of the algorithm to different kinds of computers, its simplicity and elegance, etc. [10, p. 7]

The computer scientist Edsgar Dijkstra thought that in his field 'perhaps more than everywhere else, mathematical elegance is not a dispensable luxury but decides between success and failure' [5]. In his various writings, he did not seem to make a distinction between beauty and elegance, and he subsumed computer program elegance within mathematical elegance.

And, of course, I must quote the first line of the Zen of Python: 'Beautiful is better than ugly.' [16]

In the sciences generally, aesthetic value can certainly play a role in motivation and as a guide in formulating theories, and has been studied. I have already mentioned McAllister's attempt to formulate a philosophical account of how aesthetic judgments influence the progress of science and in particular the role of aesthetic commitments in scientific revolutions. Even if one does not agree with his conclusions, his book is an excellent starting point for reading in this area.

For the third part of *F&N*, which deals with the modern role of mathematical beauty in the physical sciences, I restrict attention to specifically *mathematical* beauty. There are other forms of beauty in the sciences, which I very briefly discuss in the section where I draw the distinction. For example, the nineteenth-century physicist Oliver Lodge praised the beauty of the vortex theory of the atom, not for any mathematical formulation, but because of its metaphysical unity: it conceived of everything as being motion in the aether: atoms are vortices, light is carried as waves.

As I noted at the end of the introduction to the third part of *F&N*, 'A comprehensive history of the aesthetics of science would be a volume in itself — and remains to be written.'

JH: This has been a many-years-long project for you. Did it start organically from the notes for

your semigroups class, as described in the *TUGboat* article [3] and simply grow? Or was what grew just the LaTeX style? For this project was there a moment of beginning?

AJC: The LaTeX style grew organically from the style I created for the lecture notes on semigroups in 2012, as did my library of Ti*k*Z macros, tricks, etc. I essentially learned Ti*k*Z while creating those lecture notes — before that, I normally used MetaPost for diagrams. Then, when I revised the lecture notes the next year, I recreated essentially every diagram, since by that time I had a better idea of how to use Ti*k*Z correctly!

I had the idea of writing a history of mathematical beauty in 2006. I had finished my Ph.D. the year before and was getting started on my research career. But I was doing a lot of reading outside of mathematics, particularly in philosophy and its history, and I had a vague idea that I wanted to write a book. I recall lighting on the idea for a history of mathematical beauty in December 2006.

I did some relevant reading over the following years, but other commitments and projects intruded, and it was only in 2016 that I started working on it seriously. And then while I was working on the section on G.H. Hardy in 2017, I realized that he had died almost 70 years earlier, so his work would enter the public domain in the European Union at the end of that year. So my attention was diverted into creating *An Annotated Mathematician's Apology* [6], which occupied a certain amount of my time until late 2018. But much of my work on that book, which includes essays on the context and legacy of the *Apology*, was incorporated into *F&N*.

When I initially thought of writing the book, I thought it might be a few hundred pages long. But this tale grew in the telling', to borrow Tolkien's phrase. I realized that there were connections not just to philosophical aesthetics, but to ethics (via arguments about the justification for mathematics), epistemology, art theory, poetry, the history of ideas more generally ... And even many individual topics turned out to be far deeper than I had initially surmised. For example, early in my reading, I came across the oft-quoted remark by P.A.M. Dirac that 'it is more important to have beauty in one's equations than to have them fit experiment.' I knew it would have to be mentioned. But when I actually looked at Dirac's writings, I found that the context of that remark made it much more nuanced, and that his attitude to beauty in the mathematical formulation of physical theories was complicated. And so the section on Dirac is over fourteen pages long.

JH: Let me close with a TeX-specific question. You have become expert in at least a lot of areas of TeX. Reflections? Surprises? What was hardest thing? What keeps you coming back? (I'm fishing here; I'm sure any reflection would be of interest to *TUGboat* readers.)

AJC: What keeps me coming back is easy: the high-quality results!

To expand on that: for me, the quality of the typography includes consistency, which is part of the motive for the form/content separation in LaTeX, and which one can extend by defining semantic macros, using uniform Ti*k*Z styles, etc. I suppose the automation in LaTeX — the table of contents, citations, indexing, etc. — is also part of this.

My use of TeX and friends has been a learning process. For example, when I created the style for the lecture notes in 2012, it was little more than a collection of calls to other packages (like `titlesec` and `titletoc` for sectioning and contents), a collection of hacks, and some code cannibalized from other packages (like code from `tufte-latex` for placing float captions in the margins). As my LaTeX programming skills improved, I gradually replaced dependencies on other packages and rewrote the cannibalized code, and started using new functionality like the LaTeX hook mechanism.

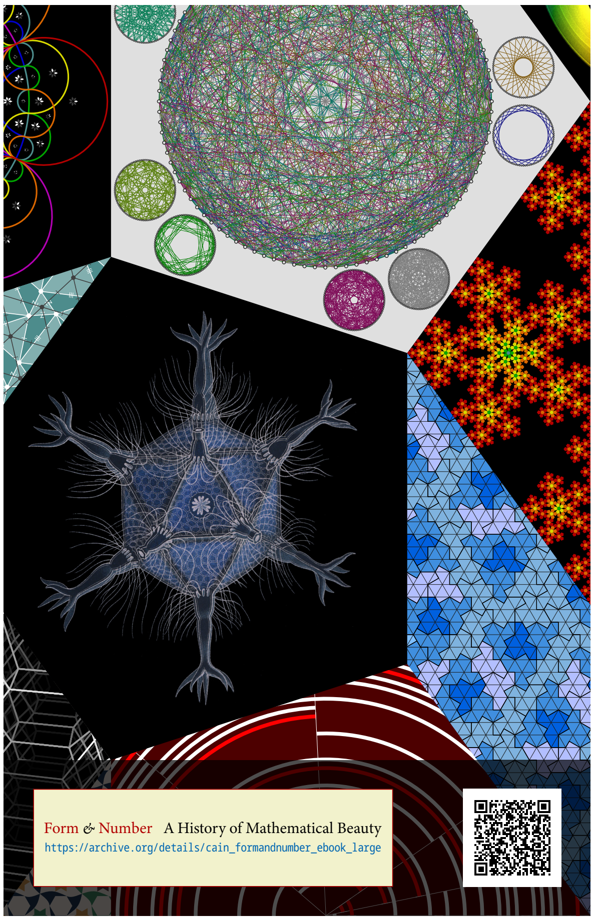
I suppose the first really complicated code that I wrote from scratch was to typeset the timeline in *F&N*. This code was the ancestor of the `timechart` package, but, although the user commands resembled those of the package, the implementation was just a collection of hacks using `etoolbox` to automate drawing Ti*k*Z. I don't recall when I first started writing with `expl3` or with Lua, but the first major code I wrote with either was to place non-floating marginal content and adjust it automatically; the LuaLaTeX `marginalia` package is a much-refined version of this code. I later completely rewrote my timeline code in `expl3` to become the `timechart` package.

I suppose that the hardest thing for me, when I started to write moderately complicated code, was the concept of expansion, simply because it required me to think differently from my previous programming endeavours. And I think that `expl3` does an excellent job of helping the coder deal with expansion. I am not sure whether I could have written `marginalia` without using `expl3`. And I would probably never have released `timechart`.

Do I have any reflections on TeX and friends? Mainly a sense of gratitude for the time and effort that many people have put into creating and maintaining them. However much time I have spent on

creating and using my own styles, I am *very* conscious of how much I have benefitted from the development of the system(s). Not just TEX and LATEX themselves, but packages like TikZ, BIBLATEX, the American Mathematical Society packages, LuaLATEX, Asymptote, MetaPost … I know that I stand on the shoulders of giants.

JH: Thank you so much for your very thoughtful answers. Thank you also for the book, which is a pleasure to read and study.



The virtual back cover of *F&N*.

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