EXERCISES IN XY-PIC

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1. INTRODUCTION

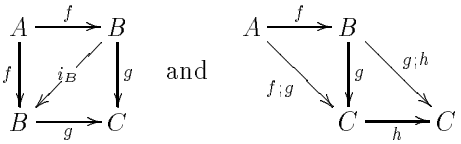
This is a collection of 'real-life' diagram typesetting problems and their solutions using the XY-pic macro package. The exercises are meant to complement the XY-pic Manual [r92:typesetting] version 2.6 (§-references in the text below refer to sections and subsections of this manual). The exercises are not much fun unless you have a running TEX system with the XY-pic macros available.[1]

We present the exercises in two groups: The first exercises are fetched from 'diagramathematics,' *i.e.*, the art of presenting lemmas, proofs, etc. of mathematics (and theoretical computer science) using diagrams. Most of these have easy solutions using XY-pic since the package was designed for just this kind of diagrams.

The second group contains complex diagrams from a variety of sources. I would like to thank David Benson, Eva Rose, Marc Andries, and Jesper Jørgensen for making their diagrams available for this collection.

2. MATHEMATICAL DIAGRAMS

Exercise 3. Typeset the axioms of category theory:

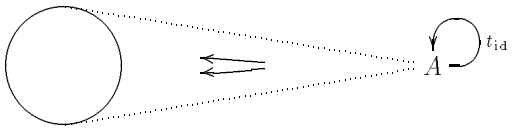


Exercise 4. Typeset the following two diagrams depicting the relation between a function and its inverse:

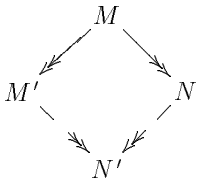
Exercise 5. Typeset a 'ligtning:'

[1] The package (including the manual) is available from any good TEX archive near you as well as by anonymous ftp from `ftp.diku.dk` [129.142.96.1] in the directory `/pub/TeX/misc/xypic.tar.Z`. You can use XY-pic with any TEX format, *e.g.*, $\mathcal{AMS}$-TEX, LATEX, or plain TEX. Installation requires METAFONT except for a generic 300 dpi laserprinter.

Exercise 6. Typeset the following characterization of the identity:

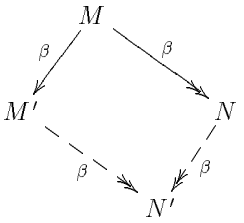


Exercise 7. Typeset the Church-Rosser theorem as formulated in [b84:lambda, p.282]:



$M, M', N, N' \in \Lambda$

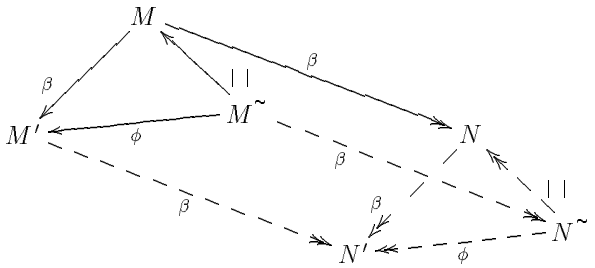
Exercise 8. Typeset the strip lemma as formulated in [b84:lambda, p.282]:



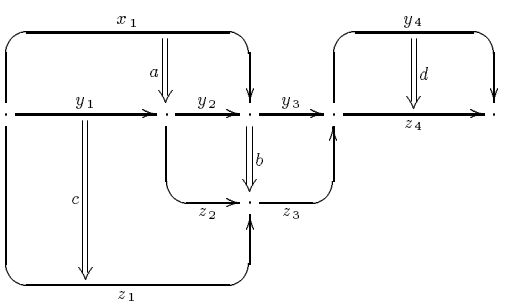
$M, M', N, N' \in \Lambda$

9. Complex Diagrams

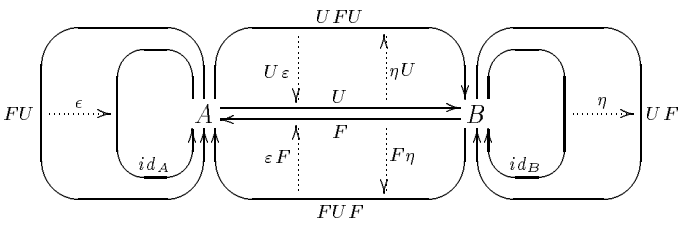
Exercise 10. Typeset this diagram used to prove the strip lemma in [b84:lambda, p.282]. *Hint*: This is difficult. It is easier if the slight lifting of the $M^{\sim}$ and $N^{\sim}$ entries is ignored.



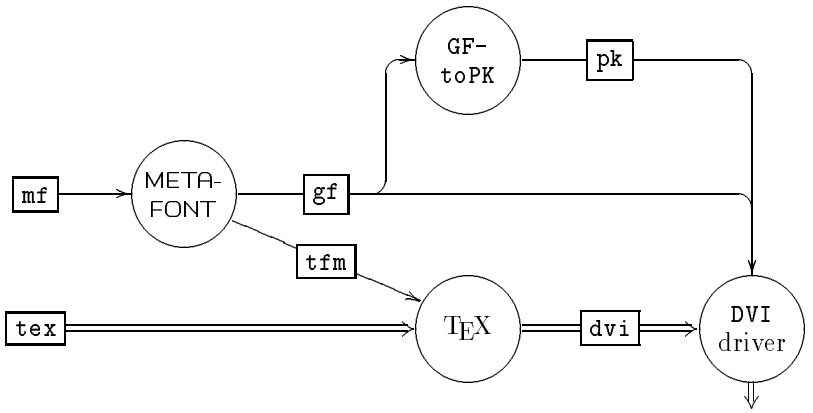
Exercise 11. Typeset David Benson's "Higher-Dimensional Graph Path:"



Exercise 12. Typeset the following categorical definition of adjoints:



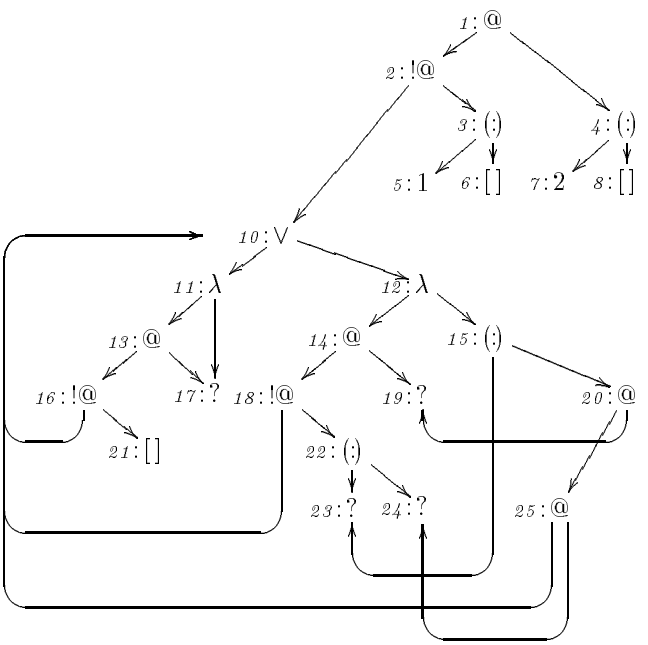
Exercise 13. Typeset the author's slide with "Standard TEX:"



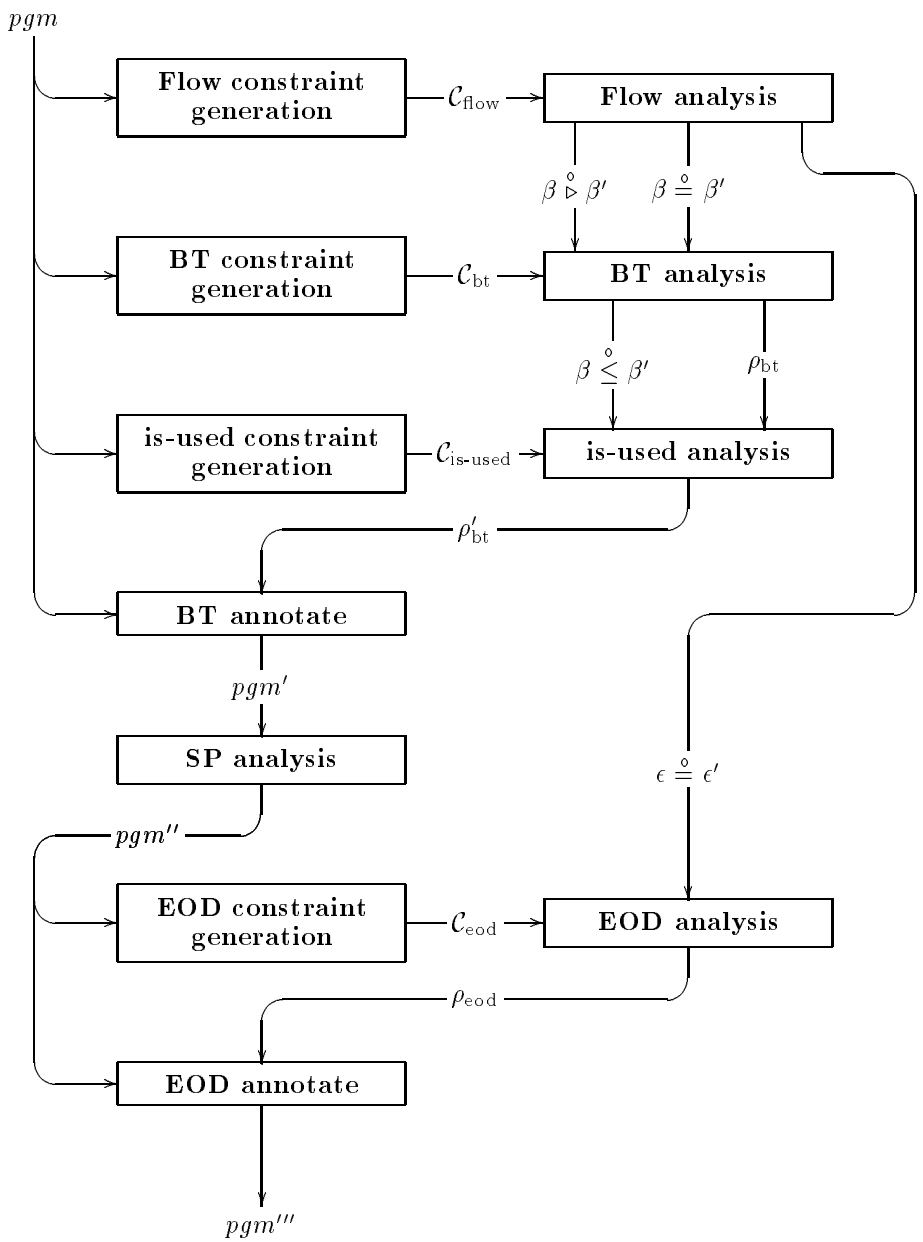
Exercise 14. Typeset the authors's "BAWL Term Graph Representation" example:

$$\left[\, [1] +\!\!+ [2] \quad \text{where} \quad \begin{array}{r} [\,] +\!\!+ ys = ys \\ (x : xs) +\!\!+ ys = x : (xs +\!\!+ ys) \end{array} \,\right]$$

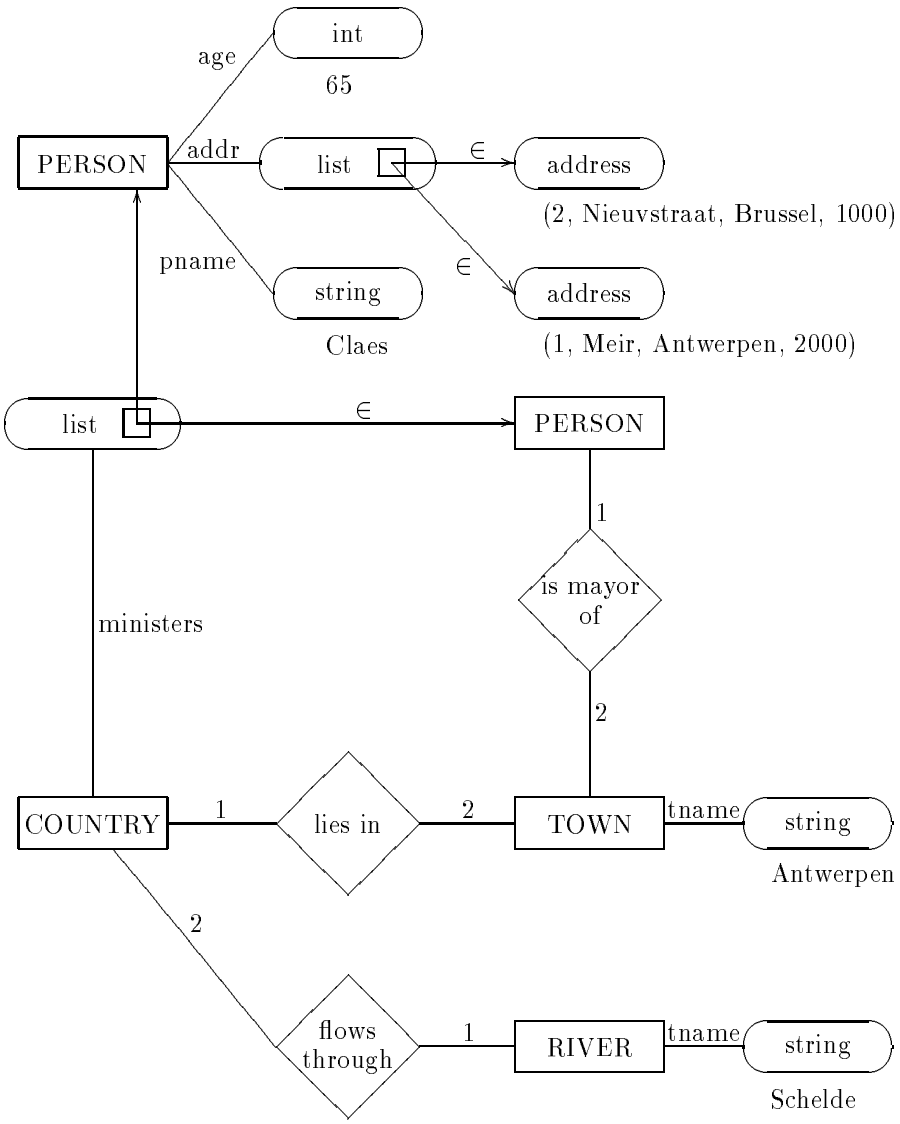
$=$



Exercise 15. Typeset Jesper Jørgensen's "Binding Time Analysis:"



Exercise 16. Typeset Marc Andries's "EER-diagram:"



17. Solutions

Solution to exercise 3. These two diagrams can be handled using only features of §1.1–1.4 combined with `\text`:

```
% axioms of category theory
$$
\diagram A \dto_f \rto^f & B \dlto|{i_B} \dto^g \\
         B        \rto_g & C \enddiagram
%
\quad\text{and}\quad
%
\diagram A \rto^f \drto_{f;g} & B \dto^g \drto^{g;h} \\
                              & C \rto_h             & C \enddiagram
$$
```

Solution to exercise 4. This illustrates the use of bending and sliding arrows of §1.5 and §2.3, respectively, and of adding an extra tail as described in §2.2. It also illustrates that the sequence of the diagram entry components is insignificant.

```
% function and inverse
$$
\diagram  x \rrtou_f && \lltod_{f^{-1}}  f(x) \enddiagram
%
\quad\text{and}\quad
%
\diagram  x \rrto<.5ex>^f && \llto<.5ex>^{f^{-1}} f(x) \enddiagram
$$
```

Solution to exercise 5. Simple by 'passing' empty entries as described in §2.5 using the target address forms described in §2.4:

```
% zig-zag
$$
\diagram A \xto '[1,0]+<-1ex,-.7ex> '[1,0]+<1ex,.7ex> [2,0] \\
         \\
         B \enddiagram
$$
```

Solution to exercise 6. The trick is to point to the right 'corners' of the large entry with the circle as described in §2.4 (except the circle which is described in §3.3). Also notice how `<>`s are used both for specifying sliding and as dimension cordinate brackets. By the way, 40pt invokes the largest available 'pure' circle.

```
% identity
$$
\diagram \squarify<40pt>{}\circled
 && \lto!<0pt,-2pt><1pt> \lto!<0pt,2pt><-1pt>
 && A \toru_{t_{\text{id}}}
      \xdotted[0,-4]+<0pt,\cH> \xdotted[0,-4]+<0pt,-\cD>
\enddiagram
$$
```

Solution to exercise 7. The thing to notice here is that we squeeze the diagram to get pleasant size of the 'diamond,' using the restyling commands of §4.3. We squeeze the rows sligthly less than the columns because entries are generally wider than their height. The solution also illustrates how diagrams may mix with other mathematics.

```
% Church-Rosser theorem
```

```
$$
\spreaddiagramrows{-.6pc}
\spreaddiagramcolumns{-.8pc}
%
\diagram & M \dlto|>>\tip \drto|>>\tip \\
 M' \drdashed|>>\tip|>\tip && N \dldashed|>>\tip|>\tip \\
         & N' \enddiagram
%
\qquad M,M',N,N' \in \Lambda
$$
```

Solution to exercise 8. This solution just squeezes the columns in order to get a proper rectangle.

```
% strip lemma
$$
\spreaddiagramcolumns{-1.2pc}
%
\diagram & M \dlto_\beta \drrto^\beta|>>\tip \\
 M' \drrdashed_\beta|>>\tip|>\tip &&& N  \dldashed^\beta|>>\tip|>\tip \\
         && N' \enddiagram
%
\qquad M,M',N,N' \in \Lambda
$$
```

Solution to exercise 10. The diagram of the proof requires some extra squeezing to make the triangles pretty. Also notice how we insert a negative thinspace `\!` in the | |-labels to get them closer to the arrow they label. The slight lifting of two of the entries is accomplished by the `\PreLift` macro: it *replaces* the `\relax` entries with the proper, 'lifted' entries (this is definitely a hack; don't despair if you didn't think of it): we explicitly name the two new entries with the same name as XY-pic uses for the 'normal' entries `"2,3"` and `"3,6"`! See §3.2 and §3.4.

```
% Proof of ``strip lemma''                                [XY-pic & -*-tex-*-]
$$
\def\PreLift"#1"#2{\save\go"#1"\go+<0ex,2ex>\Drop{#2}\go="#1"\restore}
\spreaddiagramcolumns{.4pc}
\spreaddiagramrows{.7pc}
%
\diagram
 \PreLift"2,3"{M^{^\sim}}%
 \PreLift"3,6"{N^{^\sim}}%
%
  & M \dlto_(.7)\beta \xto[1,3]^\beta |>>\tip
\\
  M' \xdashed[1,3]_\beta |>>\tip|>\tip
  && \relax \ulto_<{\!|~|}
            \llto^\phi
            \xdashed[1,3]_(.3)\beta |>>\tip|>\tip
    && N \dldashed_(.7)\beta |>>\tip|>\tip
\\
  &&& N' && \relax \uldashed_<<{\!|~|} |>>\tip|>\tip
                   \lldashed^\phi |>>\tip|>\tip
\enddiagram
$$
```

Solution to exercise 11. Good example of the path commands and how they may be used to make arrows between arows by letting the arrows pass through empty cells that may then be connected.

```
% ``Higher-Dimensional Graph Path'' (#11)                [XY-pic & -*-tex-*-]
% by David B. Benson <dbenson@yoda.eecs.wsu.edu>
$$
\diagram
& &  \ddouble_a|>\Tip   & & & \ddouble^d|>\Tip  &       \\
\cdot \xto `u"1,4" `"2,4"^{x_1} "2,4" \rrto^{y_1}
        \xto `d"4,4" `"3,4"_{z_1} "3,4"
                & \dddouble_c|>\Tip
                & \cdot \rto^{y_2} \xto `d"3,4" "3,4"_{z_2}
                & \cdot \rto^{y_3} \ddouble^b|>\Tip
                & \cdot \rrto_{z_4} \xto `u"1,7" `"2,7"^{y_4} "2,7"
                & & \cdot                \\
 & & & \cdot \xto `r"2,5"_{z_3} "2,5" & & &             \\
 & & & & & &
\enddiagram
$$
```

Solution to exercise 12. This illustrates how we can make use of a lot of empty entries of varying size to create a harmonic mixture of entries, arrows between entries, and arrows between arrows. A more technical point is that we don't need to use `\save` ... `\restore` pairs as claimed in §3.2–3.5 as long as all following XY-command in the entry start from an explicit base. Finally these demonstrate the finer points of §2.6, *e.g.*, setting the radius with `/`.

```
% Def. of Adjoint                                        [XY-pic & -*-tex-*-]
% by Eva Rose <roset@diku.dk>
%
\def\M{\squarify<1ex>{}}
\def\eps{\varepsilon}
\TurnRadius{14pt}
$$
\diagram \save  \go<-1ex,0ex>="l"\go<1ex,0ex>="r"
                \go<0em,-2pc>="d"\go<0em,2pc>="u"\restore
 &&&&&&&&
\\
 \rdotted^\epsilon|>\tip
 &
 & A \xto[0,3] <.5ex> ^U
  \go[0,0]!"l"\xto`u/10pt[0,-1]+"u"`[0,-1]
                                `[0,0]+"d"`[0,0]!"l"^{id_A}[0,0]!"l"
  \go[0,0]    \xto`u[-1,-2]`[0,-2]`[1,0]_{FU}`[0,0][0,0]
  \go[0,0]!"r"\xto`u[-1,3]`[0,3]!"l"^{UFU}[0,3]!"l"
 &\M \udotted^{U\eps}|<{\rotate\tip} \ddotted_{\eps F}|<{\rotate\tip}
 &\M \udotted_{\eta U}|>\tip \ddotted^{F\eta}|>\tip
 & B \xto[0,-3] <.5ex> ^F
  \go[0,0]!"r"\xto`u/10pt[0,1]+"u"`[0,1]`[0,0]+"d"`[0,0]!"r"_{id_B}[0,0]!"r"
  \go[0,0]    \xto`u[-1,2]`[0,2]`[1,0]^{UF}`[0,0][0,0]
  \go[0,0]!"l"\xto`d[1,-3]`[0,-3]!"r"^{FUF}[0,-3]!"r"
 &\rdotted^\eta|>\tip
 &
\\
 &&&&&&&&
\enddiagram
$$
```

Solution to exercise 13. Shows how to make arrows meet circles—this requires some computation and measuring ... The solution uses the $\mathcal{AMS}$-TEX-specific `\boxed` equivalent to `\boxit` of [1, exercise 21.3]. Also we assume that `\MF` gives the METAFONT logo!

```
% ``Standard \TeX'' diagram                                [XY-pic & -*-tex-*-]
% by Kristoffer H. Rose <kris@diku.dk>
$$
\def\CIRCLE#1{\squarify<36pt>{\Text<31pt>{#1}}\circled}
\def\T#1{\relax\ifmmode\expandafter\text\fi{\tt#1}}
\spreaddiagramcolumns{3.5pc}
\spreaddiagramrows{-1.2pc}
\TurnRadius{5pt}
%
\diagram
                &\CIRCLE{\T{GF\-to\-PK}}
                 \xto`r[2,1]|{\boxed{\T{pk}}}[2,1]
\\
 \CIRCLE{\MF}
  \save\go+<-20pt,0pt>+<-3pc,0pt>\drop{\boxed{\T{mf}}}\xto[0,0]\restore
  \save\go[0,1]\drop{}\go="gf"\restore \xline"gf"|{\boxed{\T{gf}}}
  \save\go"gf"+<-3.5pc,0pt> \xto`r+<0pc,1pc>`[-1,1][-1,1]\restore
  \save\go"gf" \xto`r[1,2][1,2]\restore
  \save\go+<18pt,-10pt>\drto+<-18pt,10pt>|{\boxed{\T{tfm}}}\restore
                &
\\
  \save\go+<-20pt,0pt>+<-3pc,0pt>\drop{\boxed{\T{tex}}}
        \xdouble[0,1]|>\Tip\restore
                &\CIRCLE{\TeX}
                  \rdouble|{\boxed{\T{dvi}}}|>\Tip
                               &\CIRCLE{\T{DVI} driver}
                                 \xdouble+<0pc,-30pt>|>\Tip
\enddiagram
$$
```

Solution to exercise 14. This is an example of a very compact diagram with mostly empty entries. It is quite simple except for the `\du` and `\dul` macros that typeset specific bent arrows, though.

This solution uses the plain TEX-specific `\displaylines` and `\eqalign` macros to align properly; this means that it is also an example of a `\diagram` ... `\enddiagram` pair that has to be enclosed in braces in order to ensure that it is precise how each `&` should be interpreted.

```
% Graph representing BAWL term [1]++[2].                [XY-pic & -*-tex-*-]
% by Kristoffer H. Rose <kris@diku.dk>
%
\def\n#1{\llap{{\sevenit#1\/}:\,}}
\def\op#1{(\mskip-.5\thinmuskip{#1}\mskip-.5\thinmuskip)}
\def\concat{\mathbin{\text{\rm+\kern-.5em+}}}
%
\def\du#1#2{%
 % #1: slidesign, #2: #rows to go down.
 \save\aftergo{\go="p",[0,0]!<#1\jot,0pt>%
  \xto`d[#2,0]+<-2.5pc,-.8pc>`"p" "p"\restore}}
%
\def\dul#1#2#3{%
 % #1: slidesign, #2: #rows to go down; #3: #columns to go left.
 \save\aftergo{\go="p",[0,0]!<#1\jot,0pt>%
  \xto`d[#2,0]+<-2.5pc,-.8pc>`[#2,-#3]+<-2.5pc,0pc> `"p" "p"\restore}}
%
\spreaddiagramrows{-1.4pc}
\spreaddiagramcolumns{-1pc}
%
```

```
$$
\displaylines{\quad
 \left\lceil\openup-\jot\eqalign{
  [1]\concat[2] \quad\text{where}\qquad
    [\,] \concat ys     &= ys \cr
    (x:xs) \concat ys   &= x:(xs \concat ys)}\right\rceil
\hfill\cr\hfill
 =\qquad
 {\diagram&&       &       &       &&\n1\@\dlto\xto[2,2]\\
   &       &       &       &       &\n2{!\@}\xto[3,-2]\drto\\
   &       &       &       &       &&\n3\op:\dlto\dto&&\n4\op:\dlto\dto\\
   &       &       &       &       &\n51&\n6[\,] &\n72&\n8[\,]\\
   &       &       &\n{10}\lor\dlto\drrto \go+<-3em,0em>="10"\\
   &       &\n{11}\lambda \dlto\ddto&&&\n{12}\lambda \dlto\drto\\
   &\n{13}\@\dlto\drto&&   &\n{14}\@\dlto\drto
                                &&\n{15}\op:\du04[3,-2]\drrto\\
 \n{16}{!\@}\dul000"10"\drto&&\n{17}{?}&\n{18}{!\@} \dul023"10" \drto
                           &&\n{19}{?}&&&\n{20}\@\ddlto\du00[0,-3]\\
   &\n{21}[\,]&    &       &\n{22}\op:\dto\drto&&  &       &\\
   &       &       &       &\n{23}{?}&\n{24}{?}
                             &&\n{25}\@ \dul-27"10"\du+3[0,-2]\\
   &       &       &       &       &       &       &\\
   &       &       &       &       &       &       &\\
   &       &       &       &       &       &       &\enddiagram}
\quad\cr}
$$
```

Solution to exercise 15. Notice the following details: The *pgm* at the top is created by an excursion and thus not an actual entry. We define `\T` for all text entries and `\L` for all labels to get them boxed and sized right, and `\_` for all subscript texts. This makes the diagram quite simple except for the winding paths that take some fiddling to get right.

```
% ``Binding Time Analysis by Constraint Solving''       [XY-pic & -*-tex-*-]
% by Jesper Jorgensen <knud@diku.dk>
%
$$
\def\T#1{\grow{\Text<3.5cm>{{\bf #1}}}\framed}
\def\L#1{\grow{#1}}
\def\_#1{_{\text{#1}}}
\def\C{{\Cal C}}
\def\O{\buildrel\circ\over}
%
\let\labelstyle=\tsize
\spreaddiagramrows{0.5cm}
\spreaddiagramcolumns{1cm}
%
\diagram
   \save\go+<-3cm,1cm>\Drop{pgm}
    \xto`d[0,0][0,0]
    \xto`d[1,0][1,0]
    \xto`d[2,0][2,0]
    \xto`d[3,0][3,0]
   \restore
%
   \T{Flow constraint generation} \rto|{\L{\C\_{flow}}}
   & \T{Flow analysis}
        \dto<-1.5cm>|{\L{\beta \O\triangleright \beta'}}
```

```
        \dto|{\L{\beta \O= \beta'}}
        \save \go!<1.5cm,0cm>
         \xto`d+<2cm,-1cm>`[3,0]+<3cm,0cm>`[3,0]`[5,0][5,0]
          |{\L{\epsilon \O= \epsilon'}} \restore
\\
   \T{BT constraint generation} \rto|{\L{\C\_{bt}}}
   & \T{BT analysis}
     \dto<-1cm>|{\L{\beta \O\leq \beta'}}
     \dto<1cm>|{\L{\rho\_{bt}}}
\\
   \T{is-used constraint generation} \rto|{\L{\C\_{is-used}}}
   & \T{is-used analysis}
     \xto`d+<-2cm,-1cm>`[1,-1]|{\L{\rho'\_{bt}}}[1,-1]
\\
   \T{BT annotate}
   \dto|{\L{pgm'}}
   &
\\
   \T{SP analysis}
   \xto`d+<-3cm,-1cm>`[1,0]+<-3cm,-2cm>|{\L{pgm''}}`[1,0][1,0]
   \xto`d+<-3cm,-1cm>`[1,0]+<-3cm,-2cm>|{\L{pgm''}}`[2,0][2,0]
   % \dto|{\L{pgm''}}
   &
\\
   \T{EOD constraint generation} \rto|{\L{\C\_{eod}}}
   % \dto|{\L{pgm''}}
   & \T{EOD analysis}
     \xto`d+<-2cm,-1cm>`[1,-1]|{\L{\rho\_{eod}}}[1,-1]
\\
   \T{EOD annotate}
   \dto
\\
   pgm'''
\enddiagram
$$
```

Solution to exercise 16. The key feature here is to create a text entry of a specific size. We do it using (portable) TEX box maneuvers; LATEX users could instead use `\makebox`.

```
% 'EER-diagram'                                          [XY-pic & -*-tex-*-]
% by Marc Andries <andries@rulwi.LeidenUniv.nl>
%
\def\TEXT<#1,#2>#3{\hbox{\setbox0=\hbox{$\Text<#1>{#3}$}%
  \dimen2=#2 \advance\dimen2-\ht0 \advance\dimen2-\dp0 \dimen2=.5\dimen2 %
  \ifdim\dimen2>0pt %
   \dimen0=\ht0 \advance\dimen0\dimen2 \ht0=\dimen0 %
   \dimen0=\dp0 \advance\dimen0\dimen2 \dp0=\dimen0 \fi
  \box0}}
%
% Special framing:
%
\def\Box#1{\TEXT<52pt,15pt>{#1}\framed}
\def\Oval#1{\TEXT<52pt,15pt>{#1}\framed<40pt>}
\def\Diamond#1{\kern5pt\TEXT<40pt,50pt>{#1}\kern5pt \save
 \go="0"+<0pt,-\cD> \xline"0"+<-\cL,0pt> \xline"0"+<\cL,0pt>
  \go"0"+<0pt,\cH>  \xline"0"+<-\cL,0pt> \xline"0"+<\cL,0pt>
\restore}
```

```
%
% Strange starting points:
%
\def\fromleftedge#1{\save \go!<-\cL,0pt> #1\restore}
\def\fromrightedge#1{\save \go!<\cR,0pt> #1\restore}
%
\def\frominnersquare#1{\quad\null \save\go+<.5\cR,0pt>
 \drop{\squarify<10pt>{}}\framed \go+<0pt,0pt> #1\restore}
%
% Miscellaneous:
%
\def\IN{\hbox{$\in$}}
\def\below#1{\save\go+<-\cL,-\cD>+<10pt,-10pt>\drop{\rlap{{#1}}}\restore}
%
\spreaddiagramrows{.5pc}
\spreaddiagramcolumns{.5pc}
\objectmargin{2pt}
\let\labelstyle=\hbox
%
$$
\diagram
                &\Oval{int}
                 \below{\quad 65}
\\
 \Box{PERSON}
 \fromrightedge{\urline+<-\cL,0pt>^(.7){age}
                \rline+<-\cL,0pt>^{addr}
                \drline+<-\cL,0pt>_(.7){pname}}
                &\Oval{list}
                 \frominnersquare{\rto+<-\cL,0pt>^(.7)\IN
                                  \drto+<-\cL,0pt>_(.7)\IN}
                                &\Oval{address}
                                 \below{(2, Nieuvstraat, Brussel, 1000)}
\\
                &\Oval{string}
                 \below{\quad Claes}
                                &\Oval{address}
                                 \below{(1, Meir, Antwerpen, 2000)}
\\
 \Oval{list}
 \frominnersquare{\xto;[-2,0]+<0pt,-9.5pt>*(0,1)
                  \rrto^\IN}%
 \ddline^{ministers}
                &               &\Box{PERSON}
\\
                &               &\Diamond{is mayor of}
                                 \uline_1 \dline^2
\\
 \Box{COUNTRY}
                &\Diamond{lies in}
                 \lline_1 \rline^2
                                &\Box{TOWN}
                                 \rline^{tname}
                                               &\Oval{string}
                                                \below{Antwerpen}
\\
                &\Diamond{flows through}
```

```
                \fromleftedge{\ulline_2}\rline^1
                              &\Box{RIVER}\rline^{tname}
                                            &\Oval{string}
                                             \below{Schelde}
\enddiagram
$$
```

References

1. Knuth, Donald E., *The TEXbook*, Addison-Wesley, 1984.

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