MathML: A Key to Math on the Web

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Abstract

As the Web gains in importance and as the needs of mathematical formalism on the Web are beginning to be met by MathML, it is an opportune time to reflect on the design decisions made by the W3C Math Working Group that resulted in the verbose markup language for transport of math on the Web that MathML turns out to be. The TEX community need not be frightened by the advent of MathML but may learn to work in the Web environment it provides.

The presentation will describe some of the key aspects of MathML and explain how it has begun to be used (by August 1999 the support for MathML may be expected to be much greater in a practical sense than it is now). Expected future developments and roles will be commented upon.

Introduction

The Math Working Group[1] of the World Wide Web Consortium[2] certainly believes that MathML, Math Markup Language (Ion and Miner, eds., 1997), is a key to math on the Web. It is a protocol for transferring mathematical knowledge, and for building tools to manipulate it, which shows great promise. There are already several implementations and tools based on MathML. Maybe its greatest strength is that MathML is a specification which is open to view and has been adopted as a Recommendation by the World Wide Web Consortium (W3C).

This contribution will discuss some of the context in which MathML has been developed; some of the design decisions that went into it will be illustrated in the live presentation.

History

In 1897, at the First International Congress of Mathematicians in Zürich (Rudio, 1898) there were not many talks.[3] Striking among their titles is "Über Pasigraphie, ihren gegenwärtigen Zustand und die pasigraphische Bewegung in Italien".[4]

The presenter, Ernst Schröder, an algebraist and logician from Karlsruhe[5] began his talk by saying that if there were any topic that really belonged at an International Conference of Mathematicians, then it was pasigraphy. He was sure that pasigraphy would take its rightful place on the agenda of all succeeding such conferences. Perhaps it is needless to say it did not.

Schröder then went on to disagree with the distinguished chair of the session, G. Peano,[6] by saying that he did not think that Leibniz's problem of providing an *algebra universalis*, a symbolic calculus for mathematics, had been solved. Peano (1858–1932) had just begun to publish, in 1894, his four-volume treatise intended to provide just that. It is here, in fact, that Peano's axioms for the natural numbers are to be found, along with axiomatizations and highly symbolic representations for much of arithmetic, algebra, geometry and calculus.

Schröder offered some of his own considerations on the topic of universal symbolics for math as part of logic. He favored a system, near that of C.S. Peirce (1867), using eighteen special symbols.

It is clear that the progress of science in general, and mathematics in particular, depends on there being a representation of its findings external to

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[1] `http://www.w3c.org/Math`, co-chairs: Angel L. Diaz (1998–2000), Patrick D. F. Ion (1997–2000), Robert L. Miner (1997–1998).

[2] `http://www.w3c.org`.

[3] I thank R. Keith Dennis for showing me his copy of the proceedings in 1997.

[4] "Pasigraphy, its present state and the pasigraphic movement in Italy" (Schröder, 1898). Pasigraphy is an artificial international language using characters (as mathematical symbols) instead of words to express ideas.

[5] Perhaps best known today from the so-called Schröder-Bernstein theorem.

[6] The Italian mathematician who can be considered leader of the pasigraphic movement in Italy.

individuals. In this way the evolving knowledge can be shared. The standard reference work on the history of mathematical notation is by Florian Cajori (1928/29).

The ontology of mathematics has changed over the centuries, so what the objects of mathematics are is by no means a given. Very roughly speaking, and seen from the present day, for the early Greeks math was geometry. Then came along a revolution started by Descartes who put forward a successful algebraic form of geometry through the introduction of coordinates. Throughout this time what numbers were was really not up for discussion although there were, for instance, differences drawn between rational and irrational numbers, and later between algebraic and transcendental numbers. At the end of the last century and the beginning of this, a new view held that all math had to be founded upon set theory and logic. And now that is seen as not all that satisfactory, so that categories, toposes, or the new developments of mathematical logic are viewed as fundamental.[7]

These remarks have been made because there is an obvious sense in which the W3C Math Working Group is just trying to create a modern form of universal symbolic language. It is not intrinsically an easy task.

The W3C — World Wide Web Consortium

The advantage of MathML mentioned above, that it is a public specification and a recommendation adopted by the W3C, stems from the sort of organisation the W3C is.

The W3C is a consortium of some 340 or so organizational members,[8] mostly commercial entities. They include big computer and software companies — such as IBM, Hewlett-Packard, Sun, Microsoft and Netscape, as well as smaller ones — and others from, for instance, the big aircraft or electricity industries — Boeing and Electricité de France, to name two. In general, the W3C is an organization joined by those who wish to understand and influence the development of the protocols and mechanisms that control the functioning of the Web. The W3C Director is Tim Berners-Lee, generally recognized as the inventor of the Web. The W3C is fully international in scope, with main offices in Cambridge, Massachusetts, USA (MITLCS), in Grenoble, France (INRIA) and in Tokyo, Japan (Keio U); it receives additional support from both DARPA and the European Commission.

The W3C forms working groups (WG) to study and produce recommendations on subjects concerning the Web. These range from HTML, HyperText Markup Language (Raggett and Jacobs, eds., 1998), the basic markup language for the Web, to PICS, the Protocol for Internet Content Specification (World Wide Web Consortium, 1997), which allows people to control access to pages based on content ratings. A list of all the concerns of the W3C is available at their main Web site mentioned above.

Particularly relevant to the present subject are the working groups centered around XML, an acronym from eXtensible Markup Language (Bray et al., 1998) and XSL/CSS (eXtensible Style Language and Cascading Style Sheets (Deach, 1999; Bos and Lie, eds., 1999; Bos et al., 1998).

Members of the W3C may request representation on any WG of interest to them (one representative and possibly one alternate at most on a WG, together entitled to one vote in deliberations) and there may be invited experts from outside the W3C in a WG, as deemed appropriate. The W3C tries to work by consensus as much as possible. The limitations on voting are there so that problems will be sorted out on technical merit rather than being subject to gross commercial pressures. There are guidelines and procedures, including voting if necessary, developed by the WG that deals with that aspect of the W3C.

SGML, HTML and XML

The specification for which the W3C is best known is HTML, presently at version 4.0. This was introduced by Berners-Lee along with the linking and transfer protocols of HTTP, Hypertext Transfer protocol (Fielding et al., 1998). In the beginning this was a language developed for a specific purpose, and not necessarily consonant with any other standards. However, it was realised that a few changes to HTML would make it a markup language obeying the principles of SGML, Standard Generalized Markup Language (International Standards Organization, 1986; van Herwijnen, 1994). SGML is an ISO international standard that describes a way of writing down document markup. It provides a very general framework, both verbose and complicated to use. There has been a lobby for some years trying to push SGML standardization as a highly desirable means for publishing production, which facilitates document re-use amongst other things.

However, the difficulties in using SGML standards in practice meant that it was unthinkable that those standards be taken straight over to the Web. True, SGML adoption would have provided a great

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[7] See the potted history in, say, a book by Godement (1998).

[8] `http://www.w3.org/Consortium/Member/List`

wealth of possibilities for Web publishing. But a Web browser would essentially incorporate a full SGML parser and, worse still, it would have to deal with the DSSSL, Document Style Semantics and Specification Language (10179:1996(E), 1996) for actual display.

To use SGML strictly, every document should be parsed against an explicit DTD (Document Type Definition) to be sure that it is a correct instance of some general class of documents and markup. This would require a level of rigor in SGML Web page programming in clear conflict with the use, and usefulness, of the Web. An SGML Web, it was feared, would rapidly age. Pages might not be produced because it would be too difficult to provide the level of correctness required for complex SGML browsers to function; also, standards would be difficult to observe in writing pages and in writing software, and the increase of entropy toward a chaotic breakdown of the Web would be rapid. Thus a cut-down, or simplified, version of SGML began to be developed for use with the World Wide Web, under the aegis of the W3C.

XML is a restricted form of SGML as a specification for markup languages. The most obvious thing unchanged is the tagging structure. As in SGML, elements of the document are marked up with tags, to form phrases like `<footag>`*element content*`<footag/>`. An element need not have any explicit content, in which case it is of the form `<bartag />`. There is a simple syntactic difference from SGML: XML elements which are not containers have tags ending with the tokens `/>`. Then again, in XML, markup has to be complete; it is not allowed, as it can be in an SGML markup specification, to shorten the markup by leaving out those end tags which can be inferred as necessary because some overall containing element has finished. For instance, paragraphs have to ended explicitly with a `</p>` and cannot be assumed to have done so just because another one has started with a `<p>`.

The next important matter for the Web is that the language design is intended to allow the processing, at least to some reasonable level, of pages without a declared DTD. An XML document can be well formed without being an instance of some DTD, although being well formed essentially means it would be possible to construct a DTD which has the document as a valid instance. So a browser which can parse XML can be allowed to process a free-form page, if XML syntax is respected.

There are many other technical differences from SGML which allow XML parsers to be much easier to. Many simplifications have come out of years of experience with SGML.

Again of great importance, perhaps more than its technical quality, is that XML is a Recommendation of the W3C. The corporate members of W3C will support its place on the Web. Browsers are being made with with XML support (e.g., by both Microsoft and Netscape). Tools, such as editors, are being produced to enable XML document creation. Associated specifications such as XSL and CSS for formatting (for an overview, see W3C Style, 1999), a DOM, Document Object Model (Apparao et al., 1998) for the Web, the RDF, Resource Description Framework (Lassila and Swick, eds., 1999) model for providing metadata,[9] as well as specifications for name spaces, linking, database queries and many other ancillary developments are underway at the W3C, using XML. The fundamental HTML is being redone as XHTML (W3C HTMLWorking Group, 1999).

The W3C Math Working Group

It has been rather paradoxical that the mathematical formulae of science have been so difficult to represent on the Web. Tim Berners-Lee, after all, was a scientist at CERN, a massive international center of physics in Geneva, when he came up with what became HTML and the Web.

In May 1997, the W3C chartered a Math Working Group to consider how to facilitate math on the Web. Originally called the HTML-Math WG, it contained representatives of diverse backgrounds. There were people from computer corporations and from publishing, computer algebra people[10] and invited experts from organizations not members[11] of the W3C.

Dave Raggett, the Math WG's W3C staff contact and author of the HTML 3.2 reference specification, was himself an early proponent of adding some math capabilities to HTML. In fact, there was some confusion over math support following HTML 3.2 (Raggett, 1997). Books appeared with sections explaining simple extensions to HTML for math that were little more than suggestions how something might be done. The extensions were not in any Recommendation accepted by the W3C. Recommendations are issued only after a long careful review process, including a period of six weeks during which

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[9] Also of importance to math on the Web, especially in education, and for which there is an interest group.

[10] IBM, Hewlett-Packard, Adobe, Elsevier Science, Wolfram Research, Maplesoft, SoftQuad, ...

[11] American Mathematical Society, Geometry Center, Stilo Technologies, (and later Design Science), ...

W3C members may vote for or against acceptance. They normally follow several working drafts, which are made available for public comment. It is a very open process. If there is enough support from the W3C members then a draft is put forward as a Recommendation. The math suggestions were dropped because the matter needed more careful consideration. But this shows the level of interest which was latent in the community.

The Math WG's original far-reaching objectives were listed as follows:

1. is suitable for teaching and scientific communication;
2. is easy to learn and to edit by hand for basic math notation, such as arithmetic, polynomials and rational functions, trigonometric expressions, univariate calculus, sequences and series, and simple matrices;
3. is well suited to template and other math editing techniques;
4. insofar as possible, allows conversion to and from other math formats, both presentational and semantic, such as TEX and computer algebra systems. Output formats may include graphical displays, speech synthesizers, computer algebra systems input, other math layout languages such as TEX, plain text displays (e.g., VT100 emulators), and print media, including braille. It is recognized that conversion to and from other notational systems or media may lose information in the process;
5. allows the passing of information intended for specific renderers;
6. supportes efficient browsing for lengthy expressions;
7. provides for extensibility, for example, through contexts, macros, new rendering schemas or new symbols; some extensions may necessitate the use of new renderers.

The above goals were endorsed by an earlier group meeting in October 1996 at the Boston W3C. Plainly they have not all been accomplished yet but much progress has been made. Point 2, ease of learning and hand editing, at least, cannot be achieved directly with a satisfactorily general and expressive markup language.

Many ideas were considered early on by the WG. TEX is clearly important for communication of mathematics nowadays. Why not just extend HTML with a TEX-like syntax? That turned out not to be simple at all. Finally, after much discussion, the WG decided that it would develop a markup language which accorded with XML. The reason is that it was realized that general acceptance of a new math specification would happen only if it embedded easily into the technology of the internet, which was coming to be dominated by XML and its relatives.

MathML 1.0 is written as an XML application, one of the first at more than a toy level. The Math WG wanted to come up with something that really met the goal of facilitating the use of math on the Web.

Many on the WG had considerable experience with TEX and could consider it as a natural paradigm for a math language for the Web. IBM too, drawing on its extensive experience with Scratchpad and then Axiom, could have itself proposed a language for math. Similarly, Wolfram Research had Mathematica, a very rich language for expressing math in ASCII characters suitable for easy Web transmission.

But there are disadvantages to such foundations for math on the Web. A specification to be generally used for math communication should be publicly developed and not proprietary. Math is almost entirely treated as public property: one cannot, in principle, patent mathematical facts. Agreement is needed from a broad spectrum of interested parties that the language provided is expressive enough for their purposes, for instance, from several symbolic algebra systems. Finally, a specification is more likely to be deployed if those who will use it have been involved in its development. It is also more likely to be realistic if people who will implement it have contributed to its development.

So the Math WG decided to fall in line with the evolving standards of the Web. This was a decision with many implications for our later work and not that easy to take. The primary goal was to create something that would be powerful, usable and adopted. And this aim has continued to drive the WG throughout.

Input considerations

Next the WG set about making an XML application. This had the unfortunate corollary that the markup developed would not directly meet the need for an easy input syntax for math on Web pages, not even for simple math.

After heavy discussion, it was eventually realized that the question of input styles was not one that could be solved initially. There are too many different communities of users of mathematical formulas to satisfy them all. Making a lower-level language, which input mechanisms could write, would encourage development of tools which could be of real help to the many not served by something as complicated as TEX or LATEX sources. The keyboard input could be accepted by applications tailored to the needs of their user communities — high school, research scientists, computer algebra users, … In fact, if a symbolic algebra system, or, for instance, Microsoft Word aided by a template input system, can write out MathML markup for equations, then their users do not have to learn a new input environment.

Several tools are already under development. The makers of Mathematica and Maple have pledged their intentions to support MathML in due course, and Design Science has already announced that its template editor, MathType 4.0, will write out MathML (Topping, 1999). IBM's techexplorer (Sutor and Dooley, 1998) already accepts and writes out some MathML in its currently released version.

Conversion of legacy documents

The WG chose a layered approach to facilitating math on the Web. Once an expressive transport protocol is agreed upon then developers can set about making their own compatible tools. In particular, one can make converters of legacy documents into editions using MathML, especially converters of material encoded in TEX. Conversion will never be completely automatic but there are several efforts underway to provide converters. For instance, two talks here will discuss the problems (Gurarie and Rahtz, 1999; Lovell, 1999), and the American Mathematical Society, Society for Industrial and Applied Mathematics and the Geometry Center have funded an on-going project to produce an appropriate tool to deal with the legacy from their TEX publishing systems.

A very simple example

One of the first formulas to try in a new math system is a quadratic equation. Looking closely at that can show quite a lot. So consider a slightly interesting equation[12] and its encoding:

$$x^2 - 79x + 1061 = 0\ . \tag{1}$$

```
\begin{equation}
    x^2 - 79 x + 1061 = 0\ .
\end{equation}
```

The LATEX is certainly simple. The equation number can be considered to have been provided by some extra-mathematical mechanism. The MathML coding of this display seems verbose by contrast:

MathML presentation coding for equation (1)

```
<math mode="display">
   <mrow>
    <mrow>
       <msup> <mi>x</mi> <mn>2</mn> </msup>
       <mo>-</mo>
       <mrow>
          <mn>79</mn>
          <mo>&InvisibleTimes;</mo>
          <mi>x</mi>
       </mrow>
       <mo>+</mo>
       <mn>1061</mn>
    </mrow>
    <mo>=</mo>
    <mn>0</mn>
   </mrow>
</math>
```

First, note that the punctuational period is not encoded here; this is intentional. The period, and its space away from the equation, should be part of the overall document, not part of the math. MathML is for the math and not for the rest of the document; TEX is a system that can handle both but then tends to mix contexts, as here. TEX is also misused when math mode is employed for special nonmathematical layouts, just as HTML table mechanisms are employed to do math. People should be inventive but there is more to the semantics of the situation than just the displayed form.[13]

A mathematical expression is enclosed within top-level tagging with `<math>`. Almost all the tag names are lowercase; XML is case-sensitive and so a choice was made for MathML. That this piece is to be a "display" as opposed to "in-line" is expressed by the value of an attribute of the `math` element, its `mode`. The values for attributes must be specified, and in quotes, in valid XML.

We see that each leaf of this parse tree, derived from the expression for a quadratic equation, is explicitly labelled as to its element type. All elements are explicitly tagged at start and finish. For math the tagging of conventionally begins with an `m` but that is really only a sop to mnemonics so that the specification is easier to create. `<mo>`, `<mi>` and `<mn>` mean math operator, identifier and number, respectively; identifiers are the sort of thing conventionally set in math italic (variables and so on); exactly what numbers are could be a problem but essentially we

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[12] This quadratic has the nice property of delivering prime numbers for integer values of x from 0 to 79; see Mollin (1997).

[13] The period which follows the element tagged with `<math mode="display">` is in the text of the document. The preferred placement of it should be expressed in the accompanying style sheet. How that is done I have not said and using this method depends on style sheets being well implemented. There is the fallback solution of including punctuation as text insertions in math, as is common with TEX.

are thinking of digit strings in some ordinary script. `<mrow>` provides a grouping construction, which allows an infix notation for operations; the operator here is made explicit with the non-printing character entity `⁢`, which is useful for speech rendering and line-breaking with continuation signs. `<msup>` denotes a special element whose children will be treated differently: the second is a superscript to the first. Of course, the parse tree (fragment), and thus the expression, is represented by the sequence of tokens read in the ordinary manner with condensation of the white space; the pretty-printing is for this exposition.

All this markup provides enough information as input to a screen renderer, or even to a composition system, that it should be able to present the equation correctly. There are many other elements with specialized functions. Let us look at the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (2)$$

which might be used to find roots of equation (1).

MathML presentation coding for equation (2)

```
<math mode="display">
  <mrow>
   <mi>x</mi> <mo>=</mo>
   <mfrac>
    <mrow>
     <mrow> <mo>-</mo> <mi>b</mi> </mrow>
     <mo>&PlusMinus;</mo>
     <msqrt>
      <mrow>
       <msup> <mi>b</mi> <mn>2</mn> </msup>
       <mo>-</mo>
       <mrow> <mn>4</mn>
        <mo>&InvisibleTimes;</mo> <mi>a</mi>
        <mo>&InvisibleTimes;</mo> <mi>c</mi>
       </mrow>
      </mrow>
     </msqrt>
     </mrow>
     <mrow>
      <mn>2</mn>
      <mo>&InvisibleTimes;</mo> <mi>a</mi>
     </mrow>
   </mfrac>
  </mrow>
</math>
```

Here we see new items: the fraction builder `<mfrac>`, the square root `<msqrt>`, a new character entity `±` (which is the plus or minus sign), and `⁢` again. To render `<mfrac>` and `<msqrt>` a routine has to do some geometry, adjusting lengths of lines to cover subexpressions and providing an appropriate size for the initial part of the square root sign. MathML provides many schemas for placement of symbols in their conventional mathematical relationships; it even extends TEX with, for instance, built-in constructions of pre-super- and pre-subscripts.

The entity `±` suggests that a renderer will have to have a whole series of fonts available to it containing representations of the characters — and there will be many of them if the whole of mathematical usage is to be supported.

The Math WG works with the STIX Project set up by the STIPUB group of publishers[14] to identify those characters in use in scientific publishing. It is hoped that arrangements will be made to find places for them in Unicode (Consortium, 1996); the Unicode Technical Committee is considering proposals in this vein. Fonts, preferably publicly and freely available, are then the next priority; STIPUB intends to support their creation, and other people, particularly Taco Hoekwater, are beginning to produce the fonts already.

Presentation and content markup

The last considerations of the previous section bring us back to the ideas mentioned at the start about the significance of signs and the semantics of formulas. So far we have seen *Presentation* markup concerned with capturing the two-dimensional layout of formulas. MathML, because of the strong interests of some of the Math WG members in symbolic computation, attempts to provide a markup language in which more of the semantics of math can be expressed; this is called *Content* markup. Thus the corresponding Content markup for the two expressions above is different (see the next column).

We see that equation (1) as a whole has been identified as an equality relationship by the surrounding element `<reln>` and its first empty child element `<eq />`. It is an equality between the result of a function application, shown by `<apply>` and a number, identified as such by `<cn>`. The Content number element `<cn>` has to be distinguished from the `<mn>` we have already met; assumptions about it may be made which may be useful to mathematics processing systems. The type of function being applied is shown by `<plus />`; it applies to a sequence of elements, two `<apply>`s and a number. The functions

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[14] STIPUB stands for "Scientific and Technical Information Publishers," a committee whose members include representatives from several learned societies and publishers. They meet from time to time to consider matters of mutual interest. STIX stands for the working group they set up to consider "Scientific and Technical Information eXchange" insofar as it concerns the characters that should be in Unicode and a set of fonts adequate to display them; see `www.ams.org/stix/`.

applied are `<power/>` and `<minus/>`, which are, respectively, binary and unary functions.

MathML content coding for equation (1)

```
<math mode="display">
<reln>
   <eq />
   <apply>
     <plus />
     <apply>
       <power />
       <ci>x</ci>
       <cn>2</cn>
     </apply>
     <apply>
       <minus />
       <apply>
         <times />
         <cn>79</cn>
         <ci>x</ci>
       </apply>
       <ci>x</ci>
     </apply>
     <cn>1061</cn>
   </apply>
   <cn>0</cn>
</reln>
</math>
```

This alternative way of marking up the expression is intended to assist computer algebra systems and search engines looking for mathematical expressions given the semantics. Content markup does not necessarily bring with it an immediately clear visually rendered form; at the very least, transformation rules need to be supplied to produce an appropriate Presentation markup. MathML has made provision for the addition of assertions about preferred presentation of Content markup and about the content semantics of Presentation markup. They can be used together but it is naturally not that easy to combine the two.

For the quadratic formula we have something similar: a relationship of equality between results of a cascade of function applications (see the next column). The entity `±` occurs again but this time made into a function label by being the content of an `<fn>` element. Otherwise, the only new items are function labels: `<divide />`, `<times />`.

In an attempt to allow the capture of much of the semantics of elementary math, some 75 or so Content elements are provided in MathML. This number may change during the revision of MathML to version 2.0, which the present Math WG is undertaking. There remains discussion as to how best to capture content semantics and how much to include. In fact, the next version has to include extension mechanisms, for presentation and content markup, and for symbols. so the users may extend the language to cover what was not thought of. The watchword thus far has been what is known in the US as K–14 math education.[15]

MathML content coding for equation (2)

```
<math mode="display">
<reln>
   <eq />
   <ci>x</ci>
   <apply>
     <divide />
     <apply>
       <fn><mo>&PlusMinus;</mo></fn>
       <apply>
         <minus />
         <ci>b</ci>
       </apply>
       <apply>
         <root />
         <apply>
           <minus />
           <apply>
             <power />
             <ci>b</ci>
             <cn>2</cn>
           </apply>
           <apply>
             <times />
             <cn>4</cn>
             <ci>a</ci>
             <ci>c</ci>
           </apply>
         </apply>
         <cn>2</cn>
       </apply>
     </apply>
     <apply>
       <times />
       <cn>2</cn>
       <ci>a</ci>
     </apply>
   </apply>
</reln>
</math>
```

Conclusion

Looking at these simple examples only scratches the surface of a markup language with about 150 primitive elements and ten times that many identified primitive character entities. The devil is in the

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[15] K–14 means two college years beyond Kindergarten through grade 12 in the US educational system. This is not an exact range specified by any educational norms. The final level may correspond in Europe to matters covered in lycée, Gymnasium or college, say.

details, as usual. Examples of formulas can be discussed much faster when speaking in front of overheads than on the printed page.

The WG is issuing a corrected version, MathML 1.01, in July 1999 and will provide a major revision and extension, MathML 2.0, by February 2000. Better integration with all the new standards from W3C will be part of version 2.0 and extensibility issues will be further addressed.

In the meantime, the promising new development is that the largest browser companies are beginning to implement MathML rendering in an essentially native way. Up to the present the best display of MathML with a browser has been with the Java plug-in WebEQ (which also has an associated equation editor), or with the W3C's testbed browser Amaya. The Math WG is also working on providing a test suite to verify compliance with the specification issued and to help those who build tools using MathML. There is a great deal of activity of MathML development in process.

Acknowledgements

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