The mayan Package and Fonts

Pablo Rosell-González
Facultad de Ciencias
Universidad Nacional Autónoma de México
pablo@ciencias.unam.mx

Abstract

The Mayans developed a complete positional numeral system. This fact facilitates calculation, particularly adding and subtracting. Their number system was a vigesimal (base 20) system, consisting of three different symbols: a dot which represents 'one', a horizontal line which represents 'five', and a cacao seed or stylized sea shell representing 'zero'. Any number between 0 and 19 can be represented with these symbols, for example, 12 is represented as two horizontal lines, one above the other, and two dots on top of them.

In this paper I will present the mayan.sty package for typesetting Mayan numbers, as well as the mayan*.mf METAFONT fonts and their PostScript Type 1 variants.

The mayan package allows the user to generate Mayan numbers by simply invoking, e.g., `\mayan{num}` which in any display math environment will typeset the number in a vertical stack using large symbols, while in paragraph mode will separate each level with a diagonal using adequate size symbols depending on the text font size.

Optional arguments are available for boxing the numbers when stacked vertically, writing the numbers in "vertical style" inside paragraphs, and writing the Arabic values beside each position.

1 Introduction

The ancient Mayan civilization developed sophisticated knowledge in sciences, particularly astronomy and mathematics. They adopted the numeral system from the Olmecs, a vigesimal (base 20) positional system, which includes a special symbol — for some a cacao seed, for others a stylized sea shell — representing zero, primarily used for avoiding ambiguity when a position has no value. (Compare with the Babylonian numeral system, see [4].)

This *discovery* (or *invention*) is of transcendental importance, because it allowed them to make complex calculations needed for the development of very advanced astronomical issues. They developed very precise solar and lunar calendars, and were able to predict the planetary positions, particularly the position of Venus, which was very important in their religion.

One of the four surviving Mayan codices, the *Dresden Codex*, is of special interest. It contains astronomical calculations of great accuracy concerning eclipse predictions and the position of Venus. Figure 1 shows fragments from page 24 of the Dresden Codex.

It can be seen that the 'zero' symbol is represented with different decorations. An important future project is to create more fonts including different designs for 'zero' and an implementation that allows a pseudo-random choice of them.

Part of the motivation for creating this package and fonts is that the Mayan numeral system is part of the curriculum in México and other Latin American countries when positional systems are studied, and there are no tools for generating them.

On the other hand, when the mayancodex and mayanstela packages are finished, they could be used in conjunction with Apostolos Syropoulos' epiolmec and similar packages and fonts for archaeological purposes.

2 The Mayan Numeral System

Given any integer $b > 1$ as a base, a *complete positional number system* consists of b symbols that represent the values $0, 1, \ldots, b-1$ and any linear arrangement where the first entry stands for b^0, the second for b^1, and in general, entry n stands for b^{n-1}.

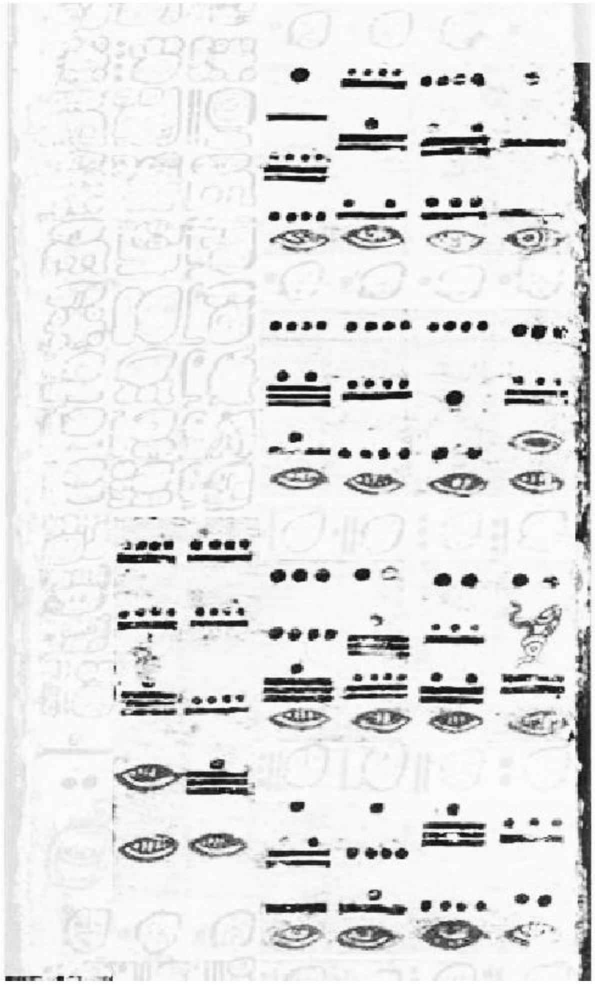


Figure 1: Fragments of page 24 of the Dresden Codex. The numbers as typically used are highlighted.

This arrangement allows us to uniquely express any number in any given base, except for the symbols used and the arrangement direction chosen. For example, if the direction of the arrangement is from left to right, the number $s_0 s_1 \ldots s_n$ where s_i takes any value from 0 to $b-1$ represents the number obtained by $\sum_{i=0}^{n} s_i \cdot b^i$.

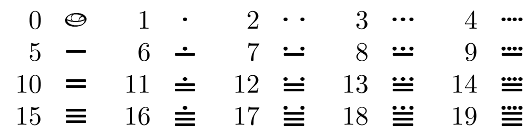
In our decimal system, the arrangement goes from right to left, thus, any number is written as $s_n s_{n-1} \ldots s_2 s_1 s_0$, where $s_i \in 0, \ldots, 9$, and i is the entry of the arrangement. For example, 876 represents the sum of $6 \cdot 10^0$ plus $7 \cdot 10^1$ plus $8 \cdot 10^2$.

The Mayans chose 20 as the base of their numeral system, probably because their solar calendar was divided in 18 months of 20 days each, plus a five day period devoted to their *infra-world* festivities.

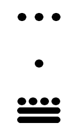
They used a dot to represent the unit and a horizontal bar to represent 5 units. The 'zero' was represented by a stylized shell and the 19 positive numbers were built according to the following rules:

1. There should be no more than four dots. (Five dots convert in a bar).
2. The dots are written above the bars.
3. The arrangement direction is vertical, from bottom to top.

From (1) and (2) we obtain, without ambiguity, the 20 required symbols:



Since the arrangement is vertical (rule 3), numbers greater than 19 are stacked. For example:

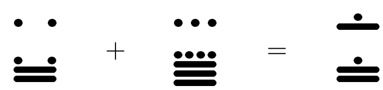


which stands for $14 \cdot 20^0 + 1 \cdot 20^1 + 3 \cdot 20^2 = 1234$.

It is worth noticing that each Mayan "digit" is built upon its predecessor. This allows us to add Mayan numbers without being conscious of which numbers they represent if we merely gather the respective levels and follow the rules mentioned above, along with the following:

4. Four bars at any level converts into a zero in the same level and a dot in the next.

For example:



where, for the bottom level, we only have to notice that by "passing" one dot of the left number to the right, and applying (4) we are done. The next level just consists of gathering 6 dots which converts into a bar (rule 1) and a dot above it (rule 2).

Which are the actual values of each of the above numbers? It doesn't matter!

In other words, the symbols that represent each "digit" are actually the value of the digit in terms of bars and dots, and not an abstract glyph — in contrast to our system.

Thus, this should be considered as an alternative method for teaching the concept of *adding* at the earliest stages of education. This way the children do not have to "translate" a symbol into a set that is in bijection with the number that the symbol represents (usually their fingers), but rather have it already written.

3 The mayan Package

The mayan.sty style file provides two relevant features: a base 10 to base 20 integer converter; and macros for different style layouts. When the user invokes `\mayan[`*opts*`]{`*num*`}`, the package performs two main tasks:

1. Convert num_{10} to num_{20}.
2. Depending on the context in which it is invoked, and the given optional arguments, call the required fonts and macros to display the result.

3.1 The Base 20 Converter

The classic method for converting num_{10} to base 20 is to recursively divide the number by 20 and keep the remainder. This is not useful because the number thus obtained would be written backwards. For example, let $num_{10} = 5246$, then

$$\begin{aligned} 5246 &= 20 \cdot 262 + 6 \\ 262 &= 20 \cdot 13 + 2 \\ 13 &= 20 \cdot 0 + 13 \end{aligned}$$

so $5246 = 6 \cdot 20^0 + 2 \cdot 20^1 + 13 \cdot 20^2$ but this way (without reversing the result) it would be printed as 6, 2, 13.

So, a different approach is used. We obtain p, the maximum power of 20 that is less than the given number, and divide it by 20^p. Print the result and keep the remainder. Apply this method recursively, until the remainder is less than 20. For example,

$$\begin{aligned} 5246 \div 20^2 &= 262 + 13 \\ 262 \div 20 &= 2 + 6. \end{aligned}$$

This time we obtain $5246 = 13 \cdot 20^2 + 2 \cdot 20 + 6$, which will be printed in the correct order.

The only tricky part is printing the middle positional zeros. For example, consider the number $320001 = 2 \cdot 20^4 + 1 \cdot 20^0$. The problem is solved by performing a subroutine which compares the remainder (r) with 20^{p-1}. If $20^{p-1}/r < 20$ do nothing; else print a zero and divide 20^{p-1} by 20 to obtain 20^{p-2}, and repeat until $20^{p-n}/r < 20$.

This approach is preferred because it avoids the need for creating extra macros to keep and then reverse the result. Instead we print the partial results 'on the fly'.

3.2 Layouts

The command `\mayan` is aware of the context from which it is invoked, i.e., it will display the Mayan number horizontally and at the same font size if invoked inside a paragraph, or vertically and in 'display' size if invoked within a display math environment (see Figure 2).

As well as the default layout, `\mayan` offers the following optional arguments:

a Write the Arabic value to the right of each position. This option is valid only when displaying the number in vertical style. It is useful for explaining positional number systems in textbooks.

b Display the number inside boxes, one for each position. Created for the same purpose as option **a**, this option is useful for emphasizing the relative values of each "digit".

h Forces horizontal style, even in display mode.

v Forces vertical style, even in horizontal mode.

Optional arguments can be given in any order.

4 The Fonts

One font family, mayanschem, is currently implemented for use in mayan, which offers simple bars and dots drawings, as well as a simple zero, mainly designed for scholarly use. These glyphs have been shown throughout the present paper.

Two more families are planned for the near future: mayancodex for "handwriting" imitating the *Dresden Codex*, and mayanstela for "engraved" designs mimicking stelae (engraved stone and stucco), like those found in Palenque and Tikal. Both will also include diverse designs for the zero found in the codices and in stelae. The user will be able to choose specific zero designs or have the system pseudo-randomly to choose a design.

5 A Complete Example

Figure 2 shows the output for the following code:

```
1   \begin{equation}
2     \mayan{5246} + \mayan{3556}
3      = \mayan{8802}
4   \end{equation}
5   since \mayan{5246} equals 5246 and
6   \mayan{3556} equals 3556, as shown below:
7   \begin{equation}
8     \mayan[ab]{5246} + \mayan[ab]{3556}
9     = \mayan[ba]{8802}
10  \end{equation}
11  {\large then $5246+3556=8802$ is
12   represented as \mayan{8802}}
```

Lines 1–4 show a displayed equation. Lines 5–6 show text in standard layout, while lines 11–12 are `\large` text. Notice that the `\mayan` macro is invoked in the same way, no matter if math or text mode, and is aware of font size.

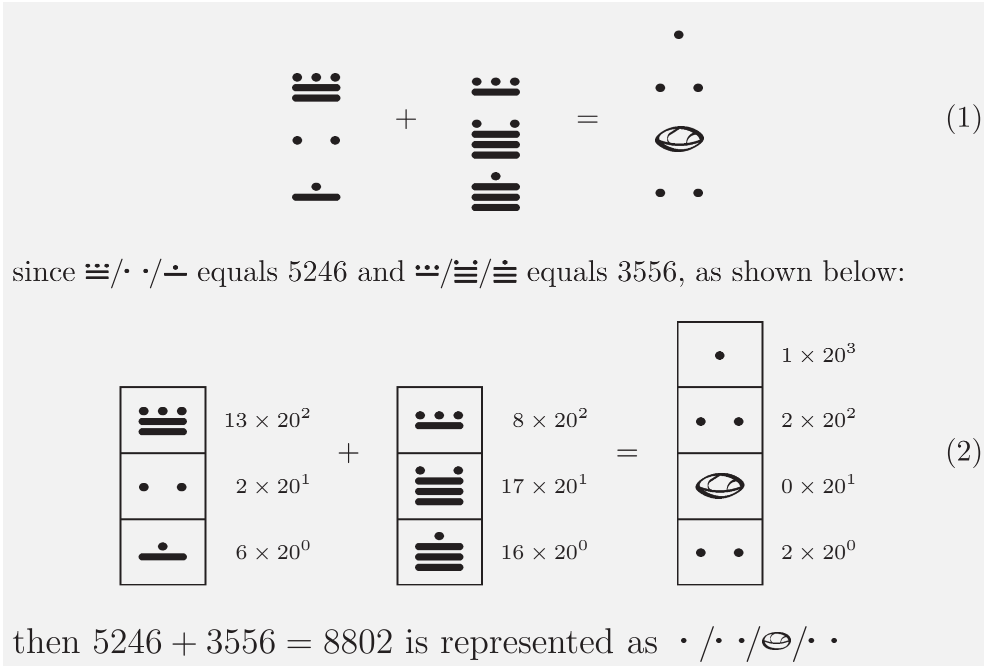


Figure 2: An example showing different layouts for `mayan`.

Lines 7–10 show the use of `\mayan` with optional parameters `a` and `b`.

References

[1] M. Goossens, F. Mittelbach, A. Samarin, *The LaTeX Companion*, Addison-Wesley, 1994.

[2] D. E. Knuth, *The METAFONTbook*, Addison-Wesley, 2000.

[3] D. E. Knuth, *The TeXbook*, Addison-Wesley, 2000.

[4] O. Ore, *Number Theory and Its History*, Dover, 1988.

[5] J. E. S. Thompson, *Códice de Dresde. Un comentario al Códice de Dresde. Libro de jeroglifos mayas*, FCE, México, 1988.