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Variation and sign tables with `tableau`

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Abstract We describe here a package, tableau.sty, created by N. Kisselhoff, very useful especially for the courses of Calculus, designed for the construction of variation and sign tables for the study of function. The package provides a new environment, based on PSTricks.

1 The commands

The main addition of the package `tableau` is a new environment, called `MonTableau`, allowing the construction of the variation/sign table, which is analogous to the classical LATEX environments, but has more flexibility.

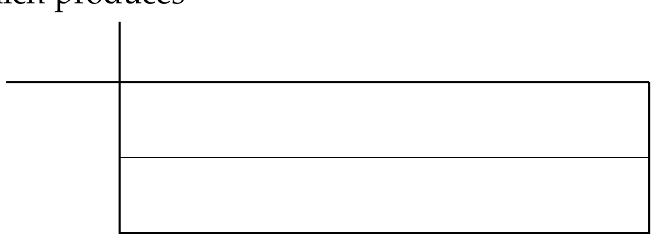
First of all, the new environment is not based on `tabular` and its relatives, but on some `PSTricks` environments. Second, unlike `tabular`, `MonTableau` is constructed one column at a time, rather than one row at a time.

A minimal table is produced by something like this:

```
\begin{MonTableau}{2}{7}{1}

\end{MonTableau}
```

which produces



Here there are three mandatory arguments:

– the first is the number of rows (beside the first one);

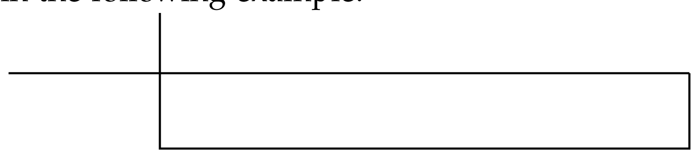
– the second is the width (in cm) of the right part of the table;

– the third is the height of the rows (in centimeters, as well).

By default, the width of the left column (the "left title" of the table) is 1.5cm. It can be modified by a `\setlength` command:

```
\setlength{\TabTitreL}{2cm}
```

as in the following example:



produced with

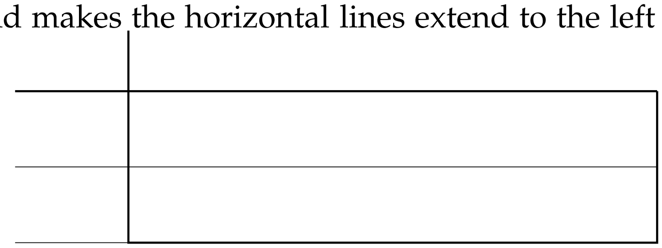
```
\setlength{\TabTitreL}{2cm}
\begin{MonTableau}{2}{7}{1}

\end{MonTableau}
```

Not everybody likes a table in which the top and left part are open as in the previous examples. Fortunately, the package provides two commands for fixing the situation. These commands have to be put inside the environment. The first command is

```
\TabTitreFerme
```

and makes the horizontal lines extend to the left columns of the table:



The second command is

```
\TabFerme
```

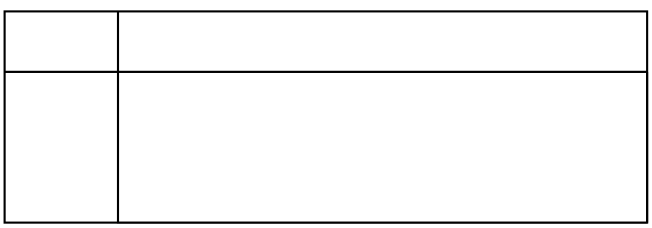
and the effect is

They can be used together, of course, and then we get:

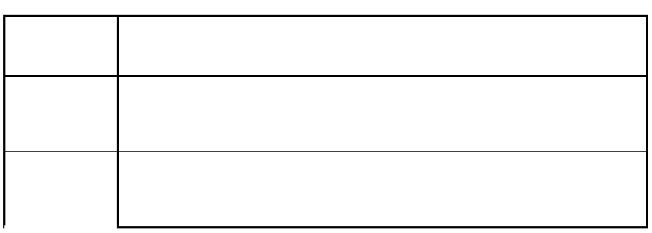
We can double the height of a row by deleting a horizontal line, with the command

```
\TabEfface{1}
```

Thus, in the previous example we can add this command (still inside the environment) and we get



The mandatory argument is, here, the number of the horizontal line. The first line is the one below the head of the table. Note that the thick lines cannot be deleted. Thus, if in the table above we try to delete the second line, what we get is:



i.e. only the left extension of the line is deleted.

Now it's time to learn how to fill the table. First of all, we have to know that, in order to put something in the table, we have to know the coordinates of the point where the object is going to be placed. We shall use two kind of coordinates, one for the left part of the table and one for the right part. For the right part, the coordinates are dimensionless. A usual table is filled row-wise. But remember, this is an unusual table, so we shall enter the information column by column. Two commands will play a crucial role in what follows.

The first simply declares that a new column is started and specifies its position:

```
\NewTableCol{position}
```

Here *position* specifies the position of the column in fractions of the total width of the right side of the column. This is the position of the *central line* of the column. The advantage of this way of specifying the position of the column, using percent rather that absolute units, is that if, for any reason, we have to change the total width of the right side of the table, we don't have to modify, also, the position of the columns.

The second command is what we need to actually put an entry in the column. The general form of the command is

```
\rTabPut[line]{position}{horizontal shift}{vertical shift}{entry}
```

In the tables of variation, we sometimes need to put a vertical line when a certain function is not defined for a given value of the independent variable. The optional argument of the command specifies what kind of line we should use. If the argument is 0, we use no line. This argument value is equivalent to the absence of the optional argument. If the value is 1, we use a dashed line, if the value is 2 the line is continuous and simple, while if the value is 3, the line is doubled. Usually (but not always!) we put no entry in a column containing a vertical line. We will return to this later.

The first mandatory argument is the reference point. More about this below, but for now, we mention that the possible reference points are those used by the PSTricks command `\rput`. Note that, unlike `\rput`, the argument is mandatoryand not optional.

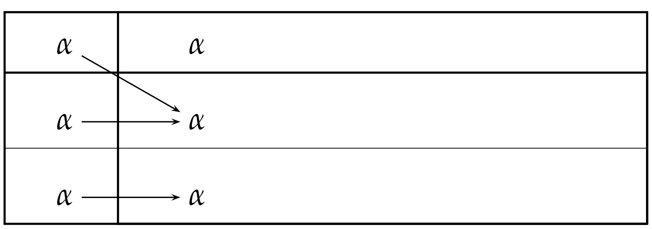
Implicitly, the column entries (or "labels") are put on the central line of the column. They can be shifted, both horizontally and vertically. The second mandatory argument indicates the horizontal shift, and the third argument specifies the vertical shift. The horizontal shift is expressed in PSTricks units (implicitly, centimeters), although the unit doesn't have to be specified. The vertical shift is expressed in fractions of the height of the row. Finally, the last mandatory argument is the entry itself. In some cases, we don't want to put anything, but the argument has to be there, even if it is empty.

The last features we would like to add to a table of variation are the arrows. To describe the command for an arrow, we have to mention, first, that to each entry of the table, we associate a node, which is described by a pair Xx. The first symbol is a capital letter indicating the column of the entry (A, B, . . .), while the second symbol is a number indicating the row of the entry. The title row corresponds to 0. Thus, a node is specified by something like $A0$, $B2$, etc.

The command for producing an arrow is very simple. You specify the origin and the extremity of the arrow, and nothing else:

```
\TabFleche{origin}{extremity}
```

where the two nodes have to be defined already. Here is a very simple example,



produced by

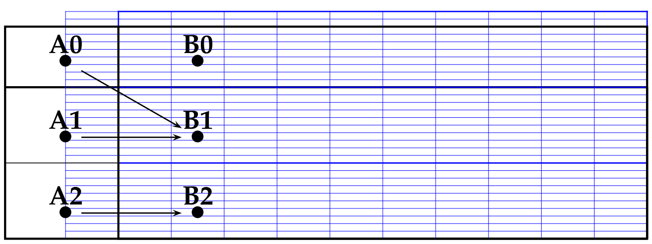
```
\begin{MonTableau}{2}{7}{1}
\TabTitreFerme
\TabFerme
\TabNewCol{-.10}
\rTabPut{B}{0}{.25}{$\alpha$}
\rTabPut{B}{0}{.25}{$\alpha$}
\rTabPut{B}{0}{.25}{$\alpha$}
\TabNewCol{.15}
```

```
\rTabPut{B}{0}{.25}{$\alpha$}
\rTabPut{B}{0}{.25}{$\alpha$}
\rTabPut{B}{0}{.25}{$\alpha$}
\TabFleche{A0}{B1}
\TabFleche{A1}{B1}
\TabFleche{A2}{B2}
\end{MonTableau}
```

If you are not sure which are the nodes on your table, you can see them by using the command

```
\TabShowLabelOn
```

that has to be placed inside the `MonTableau` environment, otherwise it will cause errors. The effect of this command is that in the table the entries are not shown, only the arrows and the nodes. For instance, in the previous example, if we want to see the nodes, the result will be:



Beware of the fact that a command `\TabShowLabelOn` will affect all the subsequent tables, not only the current one. To prevent this, you may use the command

```
\TabShowLabelOff
```

after the `\end{MonTableau}` command.

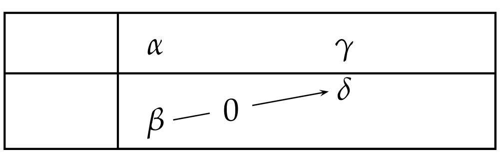
A last trick that may be used in this new environment is the command `\TabZ`. Actually, the complete form is `\TabZ[entry]`. If there is no optional argument, the command just puts a 0 on a line or on an arrow. The optional entry is what we want to put instead of 0.

Here is the first example:

produced with

```
\begin{MonTableau}{2}{7}{1}
\TabTitreFerme
\TabFerme
\TabNewCol{-.10}
\rTabPut{B}{0}{.25}{$\alpha$}
\rTabPut{B}{0}{.25}{$\alpha$}
\rTabPut{B}{0}{.25}{$\alpha$}
\TabNewCol{.15}
\rTabPut{B}{0}{.25}{$\alpha$}
\rTabPut[2]{B}{0}{.25}{\TabZ[$\alpha$]}
\rTabPut[2]{B}{0}{.25}{\TabZ}
\TabFleche{A0}{B1}
\TabFleche{A1}{B1}
\TabFleche{A2}{B2}
\end{MonTableau}
```

With `\TabZ` we can write labels in other places as well. The next example shows how to put a label on an arrow:



This example was produced by

```
\begin{MonTableau}{1}{5}{1}
\TabTitreFerme
\TabFerme
\TabNewCol{.10}
\rTabPut{B}{0}{.25}{$\alpha$}
```

```
\rTabPut{B}{0}{.25}{$\beta$}
%
\TabNewCol{.60}
\rTabPut{B}{0}{.25}{$\gamma$}
\rTabPut{B}{0}{.65}{$\delta$}
\TabFleche{A1}{B1}
\rput(.30,0.48){\TabZ}
\end{MonTableau}
```

Notice that, in this case, the command `\TabZ` is the argument of a PSTricks `\rput` command and we have to provide the coordinate of the point where the symbol is to be placed.

2 Examples

To further illustrate the use of the `MonTableau` environment we provide three examples of functions. For each we construct the table of sign and variation, and then the graph of the function. To see the LaTeX source for these examples, download the source file for this article click on the Article source files link.

In the examples below the function graphs are constructed with PSTricks commands, which are not explained here. For more on this and other graphics tools see this survey article. The graphs may also be drawn with other tools and then included with `\includegraphics`, for example.

We start with a very simple function: $f(x) = \dfrac{\ln x}{x}$. Now we shall follow the usual steps to study the variation of a function.

(i) We identify the domain of definition of the function. This is, obviously, $D = (0, \infty)$.

(ii) The function has no discontinuities in the domain.

(iii) We are looking for asymptotes.

  (a) Clearly, the ordinate axis, $x = 0$, is a vertical asymptote, because

$$\lim_{x\to 0} \frac{\ln x}{x} = -\infty.$$

(b) We can't have a left horizontal asymptote, since $-\infty$ is not in the domain on definition, but, since

$$\lim_{x\to+\infty} \frac{\ln x}{x} = 0,$$

the x-axis is a right horizontal asymptote.

(iv) We find the first derivative and the stationary points. We have

$$f'(x) = \frac{1 - \ln x}{x^2}.$$

The first derivative is defined on the entire domain of definition. We have a single stationary point, corresponding to $x = e$.

(v) The second derivative is

$$f'(x) = \frac{2\ln x - 3}{x^3}.$$

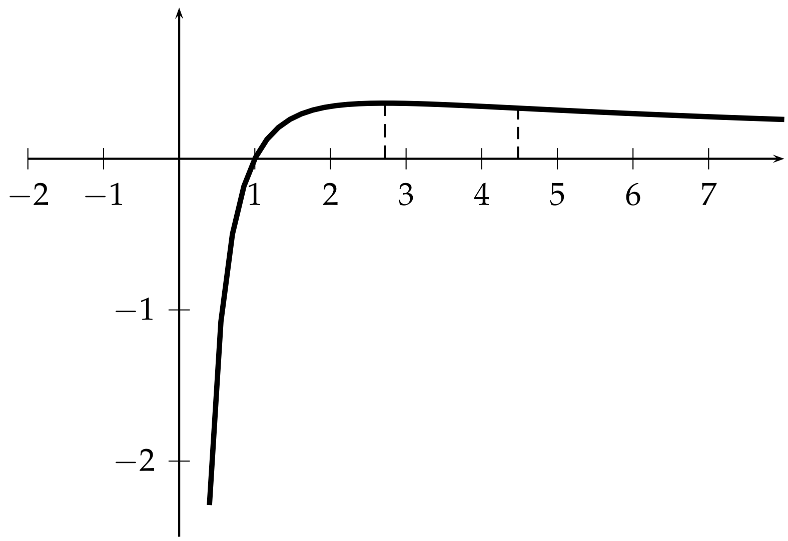
The inflexion point corresponds, of course, to $x = e^{3/2}$.

(vi) Finally, we look for the points for where the graphics cuts the coordinate axis. It is easy to see that the graphics does not intersect the y-axis, but it intersects the x-axis for $x = 1$.

Now we have all we need to construct the variation and sign table, as well as the graphics.

We start, of course, with the table.

x	0	1	e	$e^{3/2}$	$+\infty$
$f'(x)$		$+$	0	$-$	$-$
$f''(x)$		$-$	$-$	0	$+$
$f(x)$	$-\infty \longrightarrow$	$0 \longrightarrow$	$\frac{1}{e} \longrightarrow$	$\frac{3}{2}e^{-3/2} \longrightarrow$	0



The second example is a function which has singularities inside the domain. The function we chose is

$$f(x) = \frac{1}{1 + \tan x}.$$

We apply the same strategy as before:

(i) Before establishing the domain of definition, we notice that our function is periodic, with a period equal to π. Let's take, for instance, the interval $[0, \pi]$. Clearly, on this period, the function is defined on

$$D = [0, \pi] \setminus \left\{ \frac{\pi}{2}, \frac{3\pi}{4} \right\}.$$

It is enough to draw the graphics of the function on this interval.

(ii) The first derivative of the function is

$$f'(x) = -\frac{1}{1 + \sin 2x}.$$

It is strictly negative on the entire domain.

(iii) The second derivative is

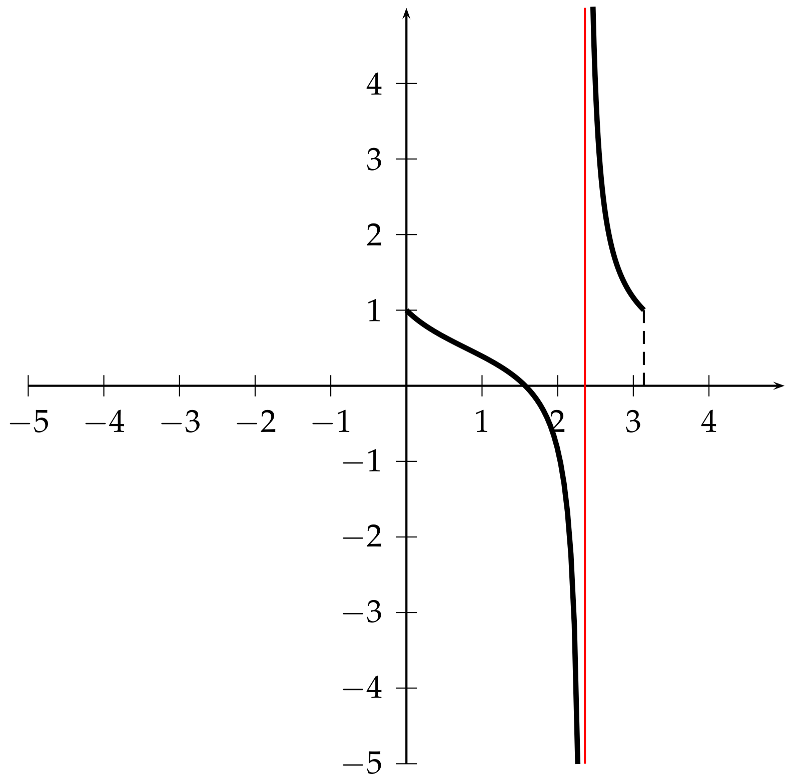
$$f''(x) = \frac{2\cos 2x}{(1+\sin 2x)^2}.$$

It vanishes for $x = \frac{\pi}{4}$.

(iv) $x = \frac{3\pi}{4}$ is vertical asymptote from both sides.

Now we construct the sign and variation table.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f'(x)$	$-$	$-$	-1	$-$ ‖ $-$	$-$
$f''(x)$	$+$	0	0	$-$ ‖ $-$	$-$
$f(x)$	1 ↘	$\frac{1}{2}$ ↘	0 ↘	$-\infty$ ‖ $+\infty$ ↘	1



The last example is a function with a more complicated domain of definition. Namely,

$$f(x) = \frac{1}{2}\sqrt{x^2 - 1}.$$

We repeat, of course, the algorithm.

(i) The domain of definition is

$$D = (-\infty, -1] \cup [1, +\infty).$$

(ii) The first derivative is

$$f'(x) = \frac{x}{2\sqrt{x^2 - 1}}.$$

The derivative does not vanish, is strictly negative for $x < -1$ and strictly positive for $x > 1$. Moreover, we have

$$f'(-1) = \lim_{x \nearrow -1} f'(x) = -\infty \quad \text{and} \quad f'(1) = \lim_{x \searrow 1} f'(x) = +\infty.$$

The function has two points of minimum, for $x = \pm 1$.

(iii) The second derivative is

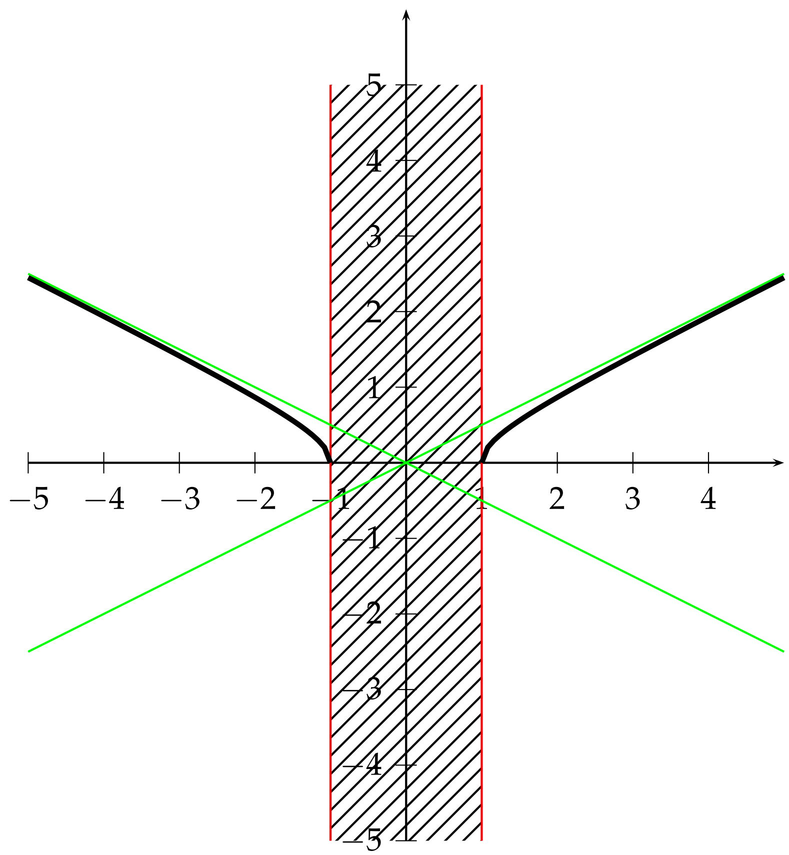
$$f''(x) = -\frac{1}{2\sqrt{x^2 - 1}}.$$

It doesn't vanish and it is strictly negative on the entire domain.

(iv) The graph has two oblique asymptotes:

$$y = \pm\frac{1}{2}x.$$

x	$-\infty$		-1		1		$+\infty$
$f'(x)$		$-$				$+$	
$f''(x)$		$-$				$-$	
$f(x)$	$+\infty$	↘	0		0	↗	$+\infty$



References

[1] N. Kisselhoff – *Package 'tableau'*, 2005, available from CTAN