Contents

List of Figures

Preface

Reasons for writing this book

Our objective here is to bring together, in one book, a number of topics in mathematics, and present them in a clear step-by-step manner. The topics are presented, where possible, in order of increasing difficulty, so allowing a natural continuity of study within the single volume.

Who can benefit from this book

The book aims to help those students who at some stage in their studies have said to their teacher or lecturer "Could you go over that step again, please?" Every attempt is made to present the material in a way that should be understandable and enjoyable to students, and also to many mainstream professionals who either wish to consolidate their earlier understanding, or take up the subject as a fresh interest - perhaps even as a retirement hobby. At all times, emphasis is placed on the understanding of the principles.

Pre-requisite knowledge

Some experience of elementary algebra, trigonometry and geometry would be an advantage.

Arrangement of material within the book

The chapters are arranged mostly with the correct progression of study in mind. However, this is not always possible, for example, fundamental algebra and the study of matrices and determinants are difficult to conveniently separate; yet, for clarity, they do need to be studied separately. Occasionally, some very elementary steps are included in the analysis to avoid later misunderstandings. There are many dedicated remarks made throughout the book. These are intended in most instances to broaden the perspective, by relating the current detail being studied to some more general concept.

Dr A K Hannaby
30^{th} September 2011

Chapter 1

Introduction: number, algebra and complex numbers

1.1 *Counting systems*

1.1.1 *The structure of the Arabic and Roman counting systems*

With surround

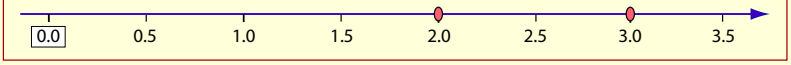

Fig 1. A double solution on a directed number line

Without surround

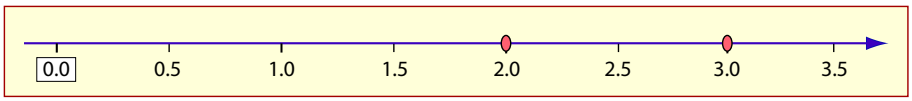

Fig 2. A double solution on a directed number line

The following tables compare and contrast two systems of counting - the Arabic and the Roman - up to twenty.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Table 1. Counting up to twenty, set out on two rows - Arabic style

Appendices

Appendix A

The binomial approximation to the exponential function

1.1 *The binomial approximation to the exponential function*

The following is a very special case of the binomial expansion...

$$\left(1+\frac{x}{n}\right)^n = 1 + n\left(\frac{x}{n}\right) + \frac{n(n-1)}{1.2}\left(\frac{x}{n}\right)^2 + \frac{n(n-1)(n-2)}{1.2.3}\left(\frac{x}{n}\right)^3 + \ldots$$
$$+\frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}\left(\frac{x}{n}\right)^r + \ldots + \left(\frac{x}{n}\right)^n$$

... we will be interested only in large positive integer values of n

This is how the expansion would look for $n=9$

$$\left(1+\frac{x}{9}\right)^9 = 1 + 9\left(\frac{x}{9}\right) + \frac{9(9-1)}{1.2}\left(\frac{x}{9}\right)^2 + \frac{9(9-1)(9-2)}{1.2.3}\left(\frac{x}{9}\right)^3 + \ldots$$
$$+\frac{9(9-1)\ldots(9-8+1)}{1.2\ldots 8}\left(\frac{x}{9}\right)^8 + \left(\frac{x}{9}\right)^9$$