only to the most salient to this present paper. We note that existence and uniqueness theorems have been proved in (1) for the initial-boundary value problem for the Korteweg-de Vries equation on the positive quarter-plane. We acknowledge that the inverse scattering transform method (ISM) has been applied to initial-boundary value problems on the positive quarter-plane (see, for example (3), (4), (5) and (14) for further discussion) and that (7) considers the Dirichlet problem for the Korteweg-de Vries equation on the positive quarter-plane when $u_{\mathrm{b}} = 0$. However, our aim in this present paper is to build on the results presented in the recent paper (8), within which the large-time solution to IBVP was examined in the three cases ($u_{\mathrm{i}} < 0, u_{\mathrm{i}} < u_{\mathrm{b}} \leqslant -2u_{\mathrm{i}}$), ($u_{\mathrm{i}} < 0, u_{\mathrm{b}} < u_{\mathrm{i}}$) and ($u_{\mathrm{i}} = 0, u_{\mathrm{b}} < 0$). The analysis presented in (8) illustrated a new approach, based on the method of matched asymptotic coordinate expansions, which although a formal method, appears to be more generally applicable than ISM. It should be noted that the method of matched asymptotic coordinate expansions is a powerful technique and is regarded as one of the cornerstones of applied mathematics and the methodology used is analogous to that developed in the context of reaction–diffusion equations and is elucidated in (11). We finally note that this approach has recently been used in (9), (10) and (12) to obtain the large-τ solution to two problems based on the Korteweg-de Vries equation. In particular, in (10), an initial-value problem was considered when the initial distribution had a discontinuous expansive step, while in (9) and (12) an initial-boundary value problem was considered on the negative quarter-plane.

In (8), the method of matched asymptotic coordinate expansions was used to develop the large-τ asymptotic structure of the solution to IBVP for values of u_{i} and u_{b} in the following ranges: (I) ($u_{\mathrm{i}} < 0, u_{\mathrm{i}} < u_{\mathrm{b}} \leqslant -2u_{\mathrm{i}}$), (II) ($u_{\mathrm{i}} < 0, u_{\mathrm{b}} < u_{\mathrm{i}}$) and (III) ($u_{\mathrm{i}} = 0, u_{\mathrm{b}} < 0$). In each case, the leading-order structure of the large-τ solution of IBVP for $x = O(1)(\geqslant 0)$ is a steady-state (time-independent) solution. In particular,

$$u(x,\tau) = \begin{cases} u_{\mathrm{i}} + 3u_{\mathrm{i}}\mathrm{cosech}^2\left(\frac{\sqrt{-u_{\mathrm{i}}}}{2}x + \mathrm{cosech}^{-1}\left[\frac{u_{\mathrm{b}}-u_{\mathrm{i}}}{3u_{\mathrm{i}}}\right]^{\frac{1}{2}}\right) \\ +O\left(\tau^{-\frac{3}{2}}\exp\left[-\frac{2}{3\sqrt{3}}(-u_{\mathrm{i}})^{\frac{3}{2}}\tau\right]\right), & \text{(I)}, \\ u_{\mathrm{i}} - 3u_{\mathrm{i}}\mathrm{sech}^2\left(\frac{\sqrt{-u_{\mathrm{i}}}}{2}x + \mathrm{sech}^{-1}\left[\frac{u_{\mathrm{b}}-u_{\mathrm{i}}}{-3u_{\mathrm{i}}}\right]^{\frac{1}{2}}\right) \\ +O\left(\tau^{-\frac{3}{2}}\exp\left[-\frac{2}{3\sqrt{3}}(-u_{\mathrm{i}})^{\frac{3}{2}}\tau\right]\right), & \text{(II)}, \\ -\left[\frac{x}{2\sqrt{3}} + \frac{1}{\sqrt{-u_{\mathrm{b}}}}\right]^{-2} + o(1), & \text{(III)}, \end{cases}$$

as $\tau \to \infty$ with $x = O(1)$. We note that the rate of convergence to the steady state for values of u_{i} and u_{b} in ranges (I) and (II) is exponential in τ as $\tau \to \infty$, being $O\left(\tau^{-\frac{3}{2}}\exp\left[-\frac{2}{3\sqrt{3}}(-u_{\mathrm{i}})^{\frac{3}{2}}\tau\right]\right)$. IBVP has been considered in (13), where exact and approximate solutions are found. The numerical and approximate results presented for values of u_{i} and u_{b} in the ranges (I) and (II) were in agreement with the detailed analysis presented in (8) and summarised above. However, we note that parameter range (III) was not considered in (13), but this steady-state solution is well known in the literature (see, for example (2)).

In this paper, we use the method of matched asymptotic coordinate expansions to develop the large-τ asymptotic structure of the solution to IBVP for the parameter ranges (IV) ($u_{\mathrm{i}} > 0, u_{\mathrm{b}} < 0$), (V) ($u_{\mathrm{i}} > 0, u_{\mathrm{b}} = 0$) and (VI) ($u_{\mathrm{i}} > 0, 0 < u_{\mathrm{b}} < u_{\mathrm{i}}$). The large-$\tau$ asymptotic structure of solution to IBVP is in each case obtained by careful consideration of the asymptotic structures as $\tau \to 0$ ($0 < x < \infty$) and as $x \to \infty$ ($\tau \geqslant O(1)$). As this paper builds on the analysis presented in (8),