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Phase control of excitable systems

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Abstract. Here we study how to control the dynamics of excitable systems by using the *phase control* technique. Excitable systems are relevant in neuronal dynamics and therefore this method might have important applications. We use the periodically driven FitzHugh–Nagumo (FHN) model, which displays both spiking and non-spiking behaviours in chaotic or periodic regimes. The phase control technique consists of applying a harmonic perturbation with a suitable phase ϕ that we adjust in search of different behaviours of the FHN dynamics. We compare our numerical results with experimental measurements performed on an electronic circuit and find good agreement between them. This method might be useful for a better understanding of excitable systems and different phenomena in neuronal dynamics.

Contents

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1. Introduction *a*

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The paradigmatic FitzHugh–Nagumo (FHN) system [1] has been broadly investigated in the past. In spite of its mathematical simplicity, many effects observed in neuronal cells are qualitatively contained in it [2]. In particular, the periodically driven FHN system has been used to investigate the effects of external perturbations on the generation of electrical pulses in neurons. More precisely, the periodically driven FHN model has been used recently to investigate the role of noise in the encoding process, i.e. in the relation between the dynamical response of the FHN neuron model and the external driving [3, 4].

Although this is an important issue from a neuronal dynamics point of view, there is a lack of inquiries on how the input–output relation could be controlled by an external perturbation. A number of techniques have been proposed to control the dynamics of different models related with the FHN system [5]–[8]. Considering that the periodically driven FHN presents chaotic dynamics, the control of this system can be tackled from a chaos control point of view [9]. The methods to control chaos can be classified into *feedback* and *nonfeedback methods* [10, 11]. Feedback methods, like the paradigmatic methods described in [12, 13], stabilize one of the unstable orbits that lies in the chaotic attractor by applying small state-dependent perturbations to the system. However, for experimental implementations, these methods require the application of a fast and adequate response to the system that sometimes might not be available. In such cases, nonfeedback methods are more useful.

An important class of nonfeedback methods is that based on applying a harmonic perturbation either to some of the parameters of the system or as an additional driving; and its effectiveness was discussed numerically and experimentally in different works [14, 15]. These types of nonfeedback methods have been mainly used to suppress chaos in periodically driven dynamical systems. A paradigmatic family of systems of this type are nonlinear oscillators, whose equations can be written as

$$\ddot{x} + \mu\dot{x} + \frac{\mathrm{d}V}{\mathrm{d}x} = F\cos(\omega t), \tag{1}$$

where μ is the damping coefficient, $V(x)$ the potential function and $F\cos(\omega t)$ an external periodic driving. Depending on the potential $V(x)$ we have different kinds of oscillators. The dynamical system described in (1) can be written as a system of two coupled first-order differential equations with a periodic driving. Such is the prototype model considered in this paper, the periodically driven FHN model.

In [15], it was observed that when these types of nonfeedback methods were used to control the dynamics of a periodically driven chaotic system, the phase difference ϕ between the periodic driving and the perturbation had a great influence in the dynamical behaviour of the system. Furthermore, Qu *et al* [16] show that ϕ plays a crucial role in the global dynamics of a well-known nonlinear oscillator, the Duffing oscillator. This control technique is called *phase control of chaos* and it has been extensively explored in [17]. Besides chaos control, the phase control has been used to control crisis-induced intermittency [18] and to control escapes in open dynamical systems [19].

In this paper, we apply this method to a paradigmatic excitable system, the periodically driven FHN system, which is one of the most utilized to study the spiking activity of a neuron [2]. We show that this control method allows one to control different aspects of the dynamics of the system, that is, to tame or enhance the spiking activity as well as to control

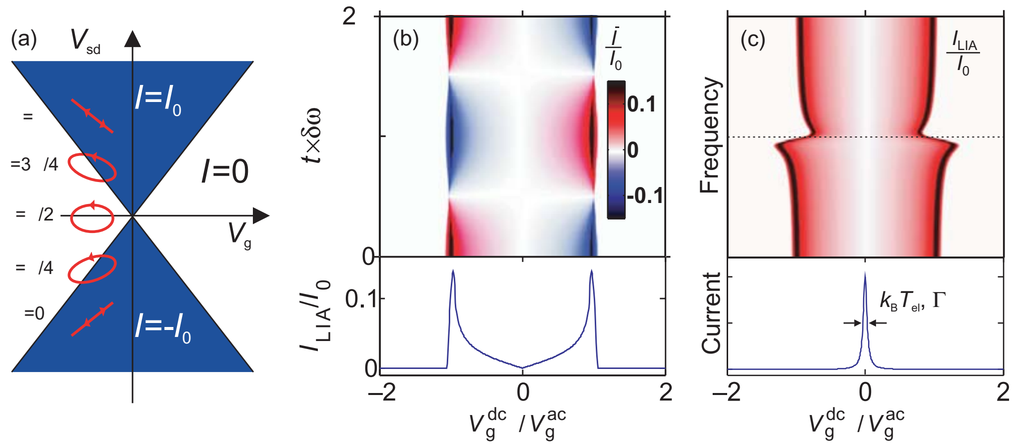

Figure 1. Numerical bifurcation diagram for the maximum of the voltage u by taking as parameter the amplitude of the external driving A. A spiking regime is observed for high values of A, whereas for low values we are in the non-spiking regime.

chaos. Thus, this work shows that the phase control method can be used to control a type of dynamics that was not considered in previous applications of this technique [15], [17]–[19]. Since the periodically driven FHN system is a paradigmatic model in neuronal dynamics, the results obtained in this work can be, in principle, generalized to higher dimensional problems such as the Hodgkin–Huxley model [20].

This paper is organized as follows. In section 2, we present the FHN model and we describe the implementation of the phase control scheme for this system. Section 3 presents numerical simulations showing that the phase control technique can both tame or enhance the spiking regime of the FHN and suppress its chaotic behaviour. Finally, we give experimental evidence of the validity and robustness of this method by implementing it in an electronic circuit, as described in section 4. Discussion and conclusions of the main results of this paper are presented in section 5.

2. Model description

The FHN system is governed by the following equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = c(-y + x - (x^3/3) + S(t)),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x - by + a, \tag{2}$$

where, in the context of neurophysiology, $x(t)$ is called the voltage variable, $y(t)$ the recovery variable, $S(t) = A\sin(\frac{2\pi}{T}t)$ is an external driving of period T and amplitude A and a, b, c are real positive constants.