TRAJECTORY NOTES

1. Tight-projectivity Lemma for SPS lattices

Let L be an SPS lattice. For an element $a \in L$, let $\mathrm{MFork}(a)$ denote the multifork at a, that is, set of all edges (prime intervals) $\mathfrak{p}$ with $1_{\mathfrak{p}} = a$. The edges in $\mathrm{MFork}(a)$ on the left and right, respectively are called the *exterior edges*; the others are called *interior* edges.

For the edges $\mathfrak{p}, \mathfrak{q}$ of L, we define a binary relation:

$\mathfrak{p}$ *swings* to $\mathfrak{q}$, in formula, $\mathfrak{p} \circlearrowright \mathfrak{q}$, if $\mathfrak{p}$ and $\mathfrak{q} \in \mathrm{MFork}(a)$ for some $a \in L$ and $\mathfrak{q}$ is an interior edge.

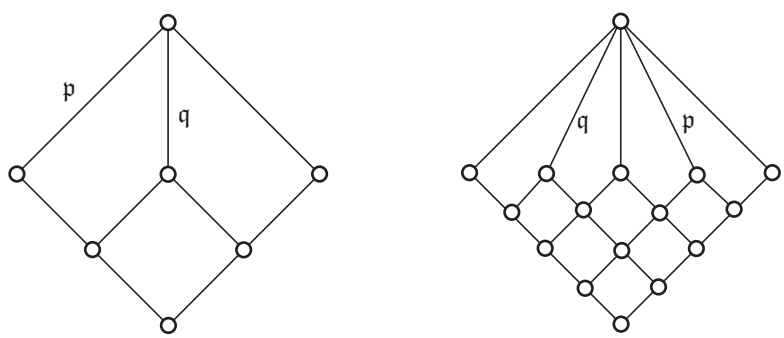

Figure 1. $\mathfrak{p}$ *swings* to $\mathfrak{q}$, two examples

Lemma 1 (Tight-projectivity Lemma for SPS lattices)**.** *Let L be an SPS lattice and let $\mathfrak{p}$ and $\mathfrak{q}$ be prime intervals in L. Then* $\mathrm{con}(\mathfrak{p}) \geq \mathrm{con}(\mathfrak{q})$ *iff there exists an edge $\mathfrak{r}$ and sequence of edges $\mathfrak{r} = \mathfrak{r}_0, \mathfrak{r}_1, \ldots, \mathfrak{r}_n = \mathfrak{q}$ such that $\mathfrak{p}$ is upper perspective to $\mathfrak{r}$, and for each $i = 0, \ldots, n-1$ $\mathfrak{r}_i$ is down perspective to $\mathfrak{r}_{i+1}$ or $\mathfrak{r}_i$ swings to $\mathfrak{r}_{i+1}$.*

Since the relations: down perspectivity and swing are transitive, two down perspectivities equal one down perspectivity and two swings equal one swing. So we can assume that down perspectivities and swings alternate.

Also note that