Math 376 Prealgebra Textbook
Chapter 1

Department of Mathematics
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Chapter 1

The Whole Numbers

Welcome you to the study of prealgebra. In this first chapter of study, we will introduce the set of natural numbers, then follow with the set of whole numbers. We will then follow with a quick review of addition, subtraction, multiplication, and division skills involving whole numbers that are prerequisite for success in the study of prealgebra. Along the way we will introduce a number of properties of the whole numbers and show how that can be used to evaluate expressions involving whole number operations.

We will also define what is meant by *prime* and *composite* numbers, discuss a number of divisibility tests, then show how any composite number can be written uniquely as a product of prime numbers. This will lay the foundation for requisite skills with fractional numbers in later chapters.

Finally, we will introduce the concept of a *variable*, then introduce equations and technique required for their solution. We will use equations to model and solve a number of real-world applications along the way.

Let's begin the journey.

1.1 An Introduction to the Whole Numbers

A *set* is a collection of objects. If the set is finite, we can describe the set completely by simply listing all the objects in the set and enclosing the list in curly braces. For example, the set

$$S = \{\texttt{dog}, \texttt{cat}, \texttt{parakeet}\}$$

is the set whose members are "dog", "cat", and "parakeet." If the set is infinite, then we need to be more clever with our description. For example, the set of *natural numbers* (or *counting numbers*) is the set

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}.$$

Because this set is infinite (there are an infinite number of natural numbers), we can't list all of them. Instead, we list the first few then follow with "three dots," which essentially mean "etcetera." The implication is that the reader sees the intended pattern and can then intuit the remaining numbers in the set. Can you see that the next few numbers are 6, 7, 8, 9, etc.?

If we add the number zero to the set of natural numbers, then we have a set of numbers that are called the *whole numbers*.

The Whole Numbers. The set

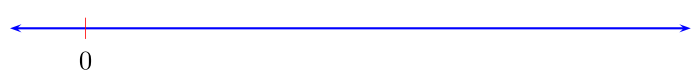
$$\mathbb{W} = \{0, 1, 2, 3, 4, 5, \ldots\}$$

is called the set of *whole numbers*.

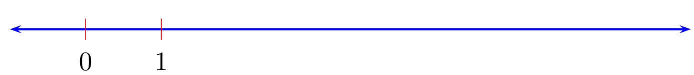
The whole numbers will be our focus in the remainder of this chapter.

Graphing numbers on the number line

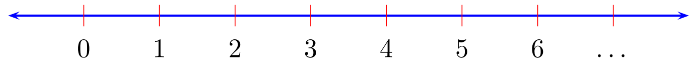
It is a simple matter to set up a correspondence between the whole numbers and points on a number line. First, draw a number line, then set a tick mark at zero.



The next step is to declare a unit length.



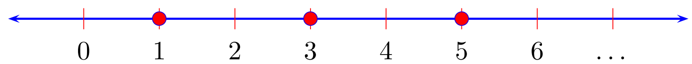
The remainder of the whole numbers now fall easily in place on the number line.



When asked to graph a whole number on a number line, shade in a solid dot at the position on the number line that corresponds to the given whole number.

EXAMPLE 1. Graph the whole numbers 1, 3, and 5 on the number line.

Solution: Shade the numbers 1, 3, and 5 on the number line as solid dots.



□

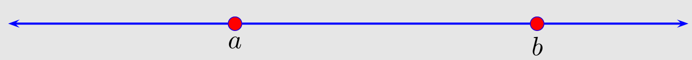
You Try It!

Use a ruler to draw a number line, then graph the whole numbers 1, 5, and 2 on the number line as solid dots.

Ordering the whole numbers

Now that we have a correspondence between the whole numbers and points on the number line, we can *order* the whole numbers in a natural way. Note that as you move to the left along the number line, the numbers get smaller; as you move to the right, the numbers get bigger. This inspires the following definition.

Ordering the Whole Numbers. Suppose that a and b are whole numbers located on the number line so that the point representing the whole number a lies to the left of the point representing the whole number b.



Then the whole number a is "less than" the whole number b and write

$$a < b.$$

Alternatively, we can also say that the whole number b is"greater than" the whole number a and write

$$b > a.$$

Comparison Property: When comparing two whole numbers a and b, only one of three possibilities is true:

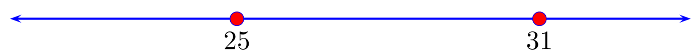
$$a < b \quad \text{or} \quad a = b \quad \text{or} \quad a > b.$$

— You Try It!

Compare the whole numbers 18 and 11.

EXAMPLE 2. Compare the whole numbers 25 and 31.

Solution: On the number line, 25 is located to the left of 31.



Therefore, 25 is less than 31 and write $25 < 31$. Alternatively, we could also note that 31 is located to the right of 25. Therefore, 31 is greater than 25 and write $31 > 25$.

Answer: $18 > 11$

□

Expanded notation

The whole numbers

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

are called *digits* and are used to construct larger whole numbers. For example, consider the whole number 222 (pronounced "two hundred twenty two"). It is made up of three twos, but the position of each two describes a different meaning or value.

2	2	2
hundreds	tens	ones

- The first two is in the "hundreds" position and represents two hundreds or 200.
- The second two is in the "tens" position and represents two tens or 20.
- The third two is in the "ones" position and represents two ones or 2.

Consider the larger number 123,456,789. The following table shows the place value of each digit.

1	2	3	4	5	6	7	8	9
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
millions			thousands			ones		

In "expanded notation," we would write

1 hundred million + 2 ten millions + 3 millions + 4 hundred thousands
+ 5 ten thousands + 6 thousands + 7 hundreds + 8 tens + 9 ones.

We read the numeral 123,456,789 as "one hundred twenty three million, four hundred fifty six thousand, seven hundred eighty nine."

Let's look at another example.

You Try It!

EXAMPLE 3. Write the number 23,712 in expanded notation, then pronounce the result.

Solution: In expanded notation, 23,712 becomes

2 ten thousands + 3 thousands + 7 hundreds + 1 ten + 2 ones.

This is pronounced "twenty three thousand, seven hundred twelve."

□

Write the number 12,572 in expanded notation, then pronounce the result.

You Try It!

EXAMPLE 4. Write the number 203,405 in expanded notation, then pronounce the result.

Solution: In expanded notation, 203,405 becomes

2 hundred thousands + 0 ten thousands + 3 thousands
+ 4 hundreds + 0 tens + 5 ones.

Since 0 ten thousands is zero and 0 tens is also zero, this can also be written

2 hundred thousands + 3 thousands + 4 hundreds + 5 ones.

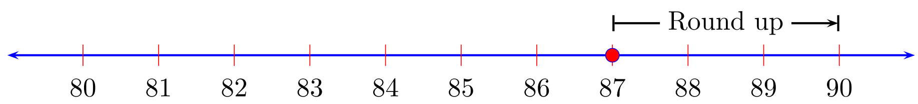
This is pronounced "two hundred three thousand, four hundred five."

□

Write the number 235,149 in expanded notation, then pronounce the result.

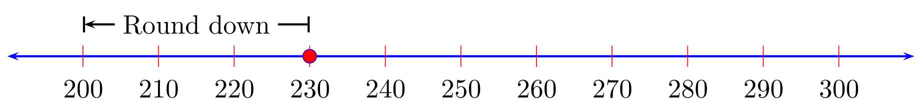
Rounding whole numbers

When less precision is needed, we round numbers to a particular place. For example, suppose a store owner needs approximately 87 boxes of ten-penny nails, but they can only be ordered in cartons containing ten boxes.



Note that 87 is located closer to 9 tens (or 90) than it is to 8 tens (or 80). Thus, rounded to the nearest ten, $87 \approx 90$ (87 approximately equals 90). The store owner decides that 90 boxes is probably a better fit for his needs.

On the other hand, the same store owner estimates that he will need 230 bags of peatmoss for his garden section.



Note that 230 is closer to 2 hundreds (or 200) than it is to 3 hundreds (or 300). The store owner worries that might have overestimated his need, so he rounds down to the nearest hundred, $230 \approx 200$ (230 approximately equals 200).

There is a simple set of rules to follow when rounding.

Rules for Rounding. To round a number to a particular place, follow these steps:

1. Mark the place you wish to round to. This is called the *rounding digit*.
2. Check the next digit to the right of your digit marked in step 1. This is called the *test digit*.
   a) If the test digit is greater than or equal to 5, add 1 to the rounding digit and replace all digits to the right of the rounding digit with zeros.
   b) If the test digit is less than 5, keep the rounding digit the same and replace all digits to the right of rounding digit with zeros.

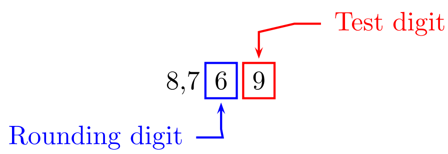
Let's try these rules with an example or two.

— You Try It!

Round 766 to the nearest ten.

EXAMPLE 5. Round the number 8,769 to the nearest ten.

Solution: Mark the rounding and test digits.



The test digit is greater than 5. The “Rules for Rounding” require that we add 1 to the rounding digit, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest ten,

$$8,769 \approx 8,770.$$

That is, 8,769 is *approximately equal* to 8,770.

er: 770

Mathematical Notation. The symbol

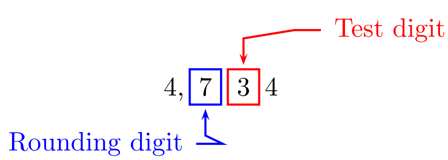
$$\approx$$

means *approximately equal*.

u Try It!

EXAMPLE 6. Round the number 4,734 to the nearest hundred.

Solution: Mark the rounding and test digits.



The test digit is less than 5. The “Rules for Rounding” require that we keep the rounding digit the same, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest hundred,

$$4,734 \approx 4,700.$$

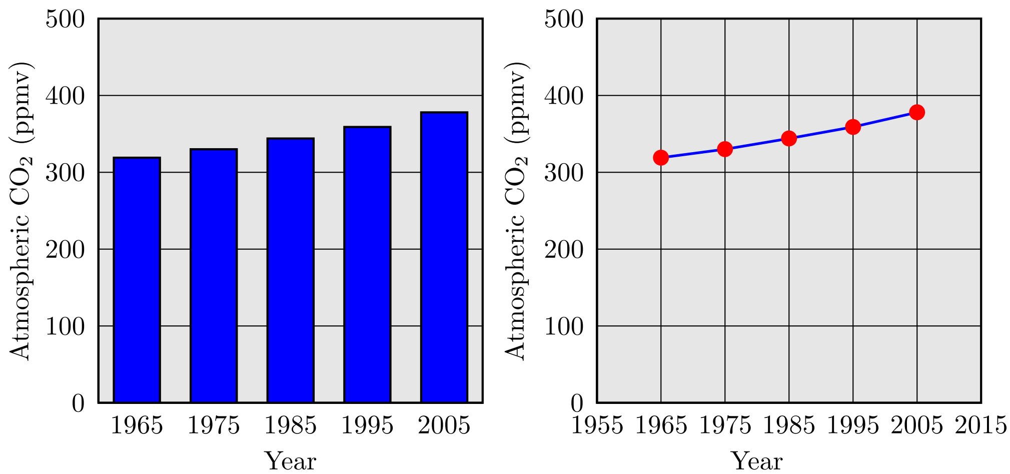
Tables and graphs

Reading data in graphical form is an important skill. The data in Table 1.1 provides measures of the carbon dioxide content (CO_2) in the atmosphere, gathered in the month of January at the observatory atop Mauna Loa in Hawaii.

Year	1965	1975	1985	1995	2005
Atmospheric CO_2	319	330	344	359	378

Table 1.1: Atmospheric CO_2 values (ppmv) derived from in situ air samples collected at Mauna Loa, Hawaii, USA.

In Figure 1.1(a), a *bar graph* is used to display the carbon dioxide measurements. The year the measurement was taken is placed on the horizontal axis, and the height of each bar equals the amount of carbon dioxide in the atmosphere during that year.



(a) Bar graph. (b) Line graph.

Figure 1.1: Using graphs to examine carbon dioxide data.

In Figure 1.1(b), a *line graph* is used to display the carbon dioxide measurements. Again, the dates of measurement are placed on the horizontal axis, and the amount of carbon dioxide in the atmosphere is placed on the vertical axis. Instead of using the height of a bar to represent the carbon dioxide measurement, we place a dot at a height that represents the carbon monoxide content. Once each data point is plotted, we connect consecutive data points with line segments.

Exercises

In Exercises 1-12, sketch the given whole numbers on a number line, then arrange them in order, from smallest to largest.

1. 2, 8, and 4

2. 2, 7, and 4

3. 1, 8, and 2

4. 0, 4, and 3

5. 0, 4, and 1

6. 3, 6, and 5

7. 4, 9, and 6

8. 2, 4, and 3

9. 0, 7, and 4

10. 2, 8, and 6

11. 1, 6, and 5

12. 0, 9, and 5

In Exercises 13-24, create a number line diagram to determine which of the two given statements is true.

13. $3 < 8$ or $3 > 8$

14. $44 < 80$ or $44 > 80$

15. $59 < 24$ or $59 > 24$

16. $15 < 11$ or $15 > 11$

17. $0 < 74$ or $0 > 74$

18. $11 < 18$ or $11 > 18$

19. $1 < 81$ or $1 > 81$

20. $65 < 83$ or $65 > 83$

21. $43 < 1$ or $43 > 1$

22. $62 < 2$ or $62 > 2$

23. $43 < 28$ or $43 > 28$

24. $73 < 21$ or $73 > 21$

---

25. Which digit is in the thousands column of the number 2,054,867,372?

26. Which digit is in the hundreds column of the number 2,318,999,087?

27. Which digit is in the hundred thousands column of the number 8,311,900,272?

28. Which digit is in the tens column of the number 1,143,676,212?

29. Which digit is in the hundred millions column of the number 9,482,616,000?

30. Which digit is in the hundreds column of the number 375,518,067?

31. Which digit is in the ten millions column of the number 5,840,596,473?

32. Which digit is in the hundred thousands column of the number 6,125,412,255?

33. Which digit is in the hundred millions column of the number 5,577,422,501?

34. Which digit is in the thousands column of the number 8,884,966,835?

35. Which digit is in the tens column of the number 2,461,717,362?

36. Which digit is in the ten millions column of the number 9,672,482,548?

37. Round the number 93,857 to the nearest thousand.

38. Round the number 56,872 to the nearest thousand.

39. Round the number 9,725 to the nearest ten.

40. Round the number 6,815 to the nearest ten.

41. Round the number 58,739 to the nearest hundred.

42. Round the number 93,146 to the nearest hundred.

43. Round the number 2,358 to the nearest ten.

44. Round the number 8,957 to the nearest ten.

45. Round the number 39,756 to the nearest thousand.

46. Round the number 24,965 to the nearest thousand.

47. Round the number 5,894 to the nearest ten.

48. Round the number 3,281 to the nearest ten.

49. Round the number 56,123 to the nearest hundred.

50. Round the number 49,635 to the nearest hundred.

51. Round the number 5,483 to the nearest ten.

52. Round the number 9,862 to the nearest ten.

---

53. According to the U.S. Census Bureau, the estimated population of the US is 304,059,724 as of July 2008. Round to the nearest hundred thousand.

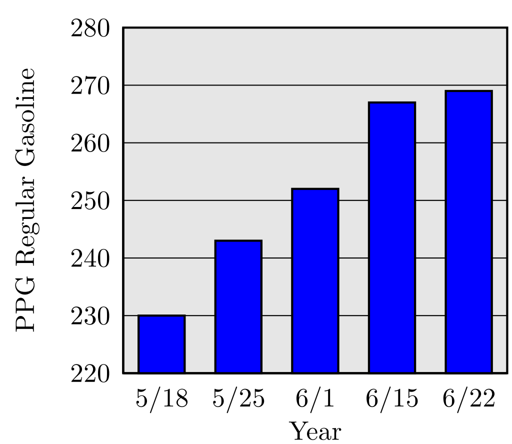
54. According to the U.S. Census Bureau, the estimated population of California is 36,756,666 as of July 2008. Round to the nearest hundred thousand.

55. According to the U.S. Census Bureau, the estimated population of Humboldt County is 129,000 as of July 2008. Round to the nearest ten thousand.

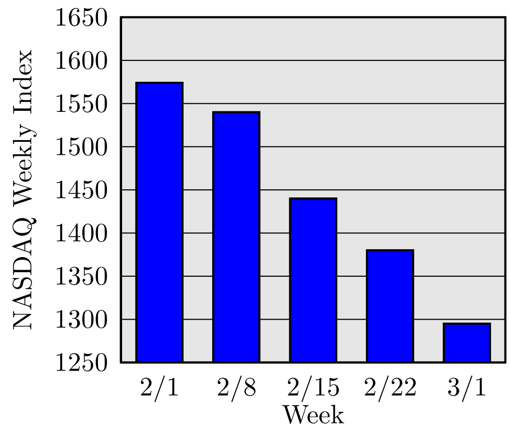
56. According to the U.S. Census Bureau, the estimated population of the state of Alasks was 686,293 as of July 2008. Round to the nearest ten thousand.

---

57. The following bar chart shows the average price (in cents) of one gallon of regular gasoline in the United States over five consecutive weeks in 2009, running from May 18 (5/18) through June 22 (6/22). What was the price (in cents) of one gallon of regular gasoline on June 1, 2009?



58. The following bar chart shows the average weekly NASDAQ index for five consecutive weeks in 2009, beginning with week starting February 1 (2/1) and ending with the week starting March 1 (3/1). What was the average NASDAQ index for the week starting February 8, 2009?



59. The population of Humboldt County is broken into age brackets in the following table. *Source: WolframAlpha.*

Age in years	Number
under 5	7,322
5-18	26,672
18-65	78,142
over 65	16,194

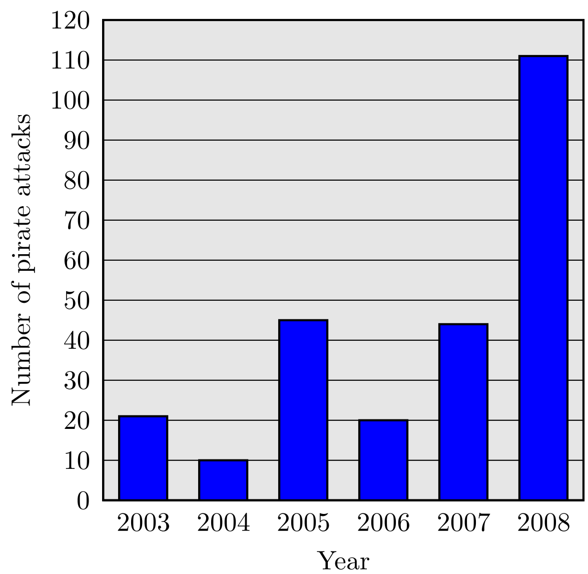
Create a bar chart for this data set with one bar for each age category.

60. The five cities with the largest number of reported violent crimes in the year 2007 are reported in the following table. *Source: Wikipedia.*

City	Violent Crimes
Detroit	2,289
St. Louis	2,196
Memphis	1,951
Oakland	1,918
Baltimore	1,631

Create a bar chart for this data set with one bar for each city.

61. The following bar chart tracks pirate attacks off the coast of Somalia.

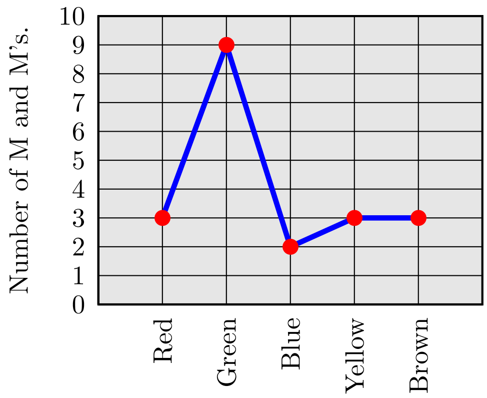


Source: ICC International Maritime Bureau, AP Times-Standard, 4/15/2009

a) How many pirate attacks were there in 2003?

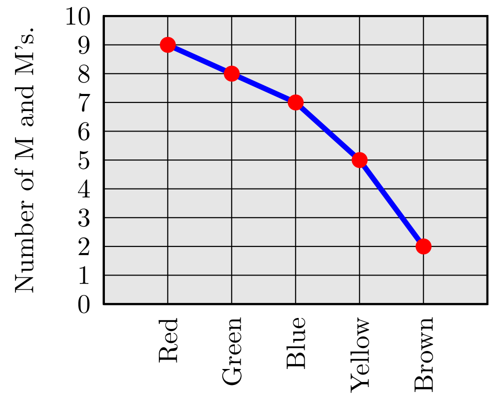
b) How many pirate attacks were there in 2008?

62. A team of students separated a small bowl of M and M's into five piles by color. The following line plot indicates the number of M and M's of each color.



How many red M and M's were in the bowl?

63. A team of students separated a small bowl of M and M's into five piles by color. The following line plot indicates the number of M and M's of each color.



How many red M and M's were in the bowl?

---

64. A team of students separated a small bowl of M and M's into five piles by color. The following table indicates the number of M and M's of each color.

Color	Number
Red	5
Green	9
Blue	7
Yellow	2
Brown	3

Create a lineplot for the M and M data. On the horizonta axis, arrange the colors in the same order as presented in the table above.

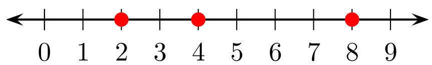
65. A team of students separated a small bowl of M and M's into five piles by color. The following table indicates the number of M and M's of each color.

Color	Number
Red	3
Green	7
Blue	2
Yellow	4
Brown	9

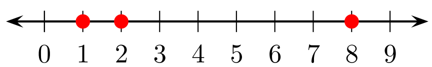
Create a lineplot for the M and M data. On the horizonta axis, arrange the colors in the same order as presented in the table above.

Answers

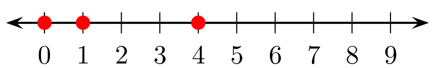
1. Smallest to largest: 2, 4, and 8.



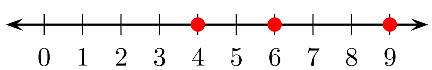
3. Smallest to largest: 1, 2, and 8.



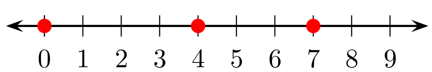
5. Smallest to largest: 0, 1, and 4.



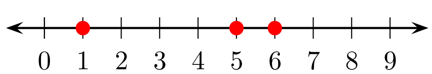
7. Smallest to largest: 4, 6, and 9.



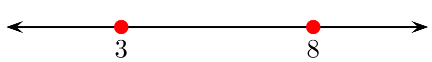
9. Smallest to largest: 0, 4, and 7.



11. Smallest to largest: 1, 5, and 6.

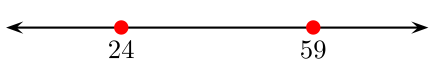


13.



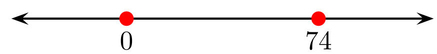
Therefore, $3 < 8$.

15.



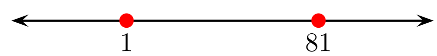
Therefore, $59 > 24$.

17.



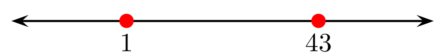
Therefore, $0 < 74$.

19.



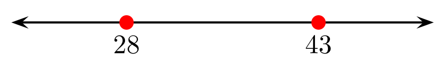
Therefore, $1 < 81$.

21.



Therefore, $43 > 1$.

23.



Therefore, $43 > 28$.

25. 7

27. 9

29. 4

31. 4

33. 5

35. 6

37. 94000

39. 9730

41. 58700

43. 2360

45. 40000

47. 5890

49. 56100

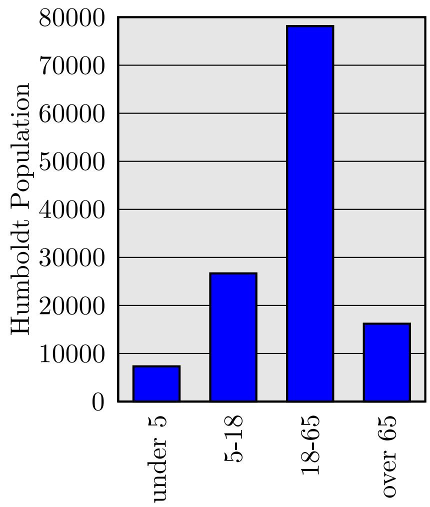
51. 5480

53. 304,100,000

55. 130,000

57. Approximately 252 cents

59.



61. a) Approximately 21

b) Approximately 111

63. 9

65.