Taylor's Theorem

Theorem and Definition: Let f be an $(n+1)$–times continuously differentiable function on an open interval $I \in \mathbb{R}$. Then for any $x, x_0 \in I$,

$$f(x) = T_{f,x_0,n}(x) + R_{f,x_0,n}(x),$$

where

$$T_{f,x_0,n}(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

is called the *Taylor polynomial of degree n at x_0* and

$$R_{f,x_0,n}(x) := \frac{(x - x_0)^{n+1}}{n!} \int_0^1 t^n f^{(n+1)}(x + t(x_0 - x))\, dt$$

is called the remainder term. (There are other formulations of the remainder term, but this one is the most useful for estimating $\sup_{x \in I} \left| R_{f,x_0,n}(x) \right|$.)