Typography

Maths = Typography?

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Introduction

This paper is written for a conference with the theme 'Hidden typography'. Broadly the author's interest is in mathematical printing and typesetting in particular. So, as is usual for academic conferences, the author's task is to persuade you, the reader, that there is some connection between the conference subject and the author's personal interest. To see if this can be done it is pertinent to ask a few questions:

* What is typography and so what is hidden typography?
* What is mathematics?
* Is there any typography in mathematics and is there any hidden typography in it?

The last of these questions is the one that is central to the subject of the conference; answers to the other two help to explain the author's answer to it. So to start with the conclusion:

* There is typography in mathematics.
* The typography in written mathematics is not hidden, it is overlooked.
* There is a strong case for saying that written mathematics is a very highly developed example of typography: it may even be possible to say 'Maths = Typography'.

What is typography?

> The art or process of setting and arranging types and printing from them. (*Concise Oxford English Dictionary, 10th edition*, 2001)

> Typography may be defined as the craft of rightly disposing printing material in accordance with specific purpose; of so controlling the type as to aid to the maximum the reader's comprehension of the text. (Stanley Morison, *First principles of typography*, 1951, CUP)

Here are two definitions of typography, one short and written for a general audience, the other longer and written for an audience wanting to know more. Both are pertinent to the purposes of this paper: it is the aspect of 'arranging' or 'rightly disposing' material 'to aid to the maximum the reader's comprehension' that will be emphasized. Some will argue that both these definitions are rather utilitarian and omit any feeling for the art and beauty that typography can bring to the printed document. However the question of beauty in mathematical typography is also addressed.

So what is *hidden* typography? In the best sense it is Beatrice Warde's crystal goblet typography: invisible or unobtrusive but making the reader's task easier and more pleasant. It is design that helps the reader to extract meaning from the written word. This is very much the sense relevant to the printing of maths. Enough people have trouble grappling with the abstraction of maths that it would not be a good idea to add typographical flourishes and quirks to its written form. Good 'crystal goblet' typography is what the complexity of maths typesetting really does need.

What is mathematics?

> The branch of science concerned with number, quantity, and space, either as abstract concepts (pure mathematics) or as applied to physics, engineering, and other subjects (applied mathematics). (*Concise Oxford English Dictionary*, 2001)

Mathematics is its own branch of science (like physics or chemistry) and comes in two forms, pure and applied. Both forms are concerned with 'number, quantity, and space', one in the abstract, one in practical terms. The graphic representation of maths then has to be able to encompass both abstract notions and practical applications if it is to be any use to mathematicians and those who use maths (physicists, engineers, etc.). It has to deal with 'number, quantity, and space'. It also has to be able to describe a whole branch of science (part will not do). As we know the result is that the printing of mathematics is challenging and specialist work largely avoided by many.

Another common view of mathematics, particularly popular with those who use maths (engineers, physicists, etc.) is that it is a *language* that is used to describe physical situations and relationships. Mathematical equations are used to describe the motion of a pendulum, the decay of radioactive waste, the flow of traffic on congested roads, and the relationship between infinitely large groups of objects in multidimensional space. Mathematics is the language that scientists (physical scientists at least) use to communicate their ideas and observations. Like mathematics in the dictionary definition

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above, the physical scientist is interested in 'number, quantity, and space'. The graphic representation of maths has to reflect this.

Having used the standard trick of looking at a dictionary definition of the subject it may be instructive to consider the popular views of its users and practitioners. A physicist uses a variety of strange machines to investigate the rules that govern the physical world and records observations as mathematical relationships. A biologist grows then experiments on living things in order to understand more about them perhaps using statistics to support arguments and observations. An engineer designs and builds machinery to exploit the discoveries of other scientists and uses approximate equations to predict how the machinery will behave. The mathematician sits and thinks and scribbles and rearranges equations on paper or blackboard. The mathematician has no machinery or plants or animals to work with. The mathematician's only prop in this populist view is the piece of paper or blackboard, or more specifically equations written on these. Mathematics is in some sense the written equations on these surfaces. If this is indeed so, then it can be appreciated that the optimal arrangement of the symbols in the equations is of some consequence.

So we have three views of mathematics: it is a branch of science dealing with the description of the abstract and real and quantity, number, and space; it is a language; and it is written equations. Properly, and not just as a result of undue deference to the majesty of the Oxford English Dictionary, it is only the first of these. But mathematics' very singular distinction is that it is dependent on its own language to communicate it. That language is only easily communicated in its written form (equations on paper or blackboard). While the individual components of an equation can be read out loud and their relative positions can be described, it is not too far-fetched to say that maths is an unpronouncable, even a silent, language. So that its written form, equations, has to be able to communicate matters of 'quantity, number, and space'.

Printing's influence on mathematics

Before pursuing the intellectual argument that written maths involves a lot of typography, it is instructive and interesting to look at the influence of printing on the development of maths. It is also instructive to see the consequences of trying to write mathematics without symbols to understand why the language of mathematics is necessary.

One of the very earliest mathematical works is the *Algebra* of Al-Khowarazimi, a ninth-century scholar in Baghdad. Florian Cajori (*A history of mathematical notation*, 1929, Open Court) quotes a translation of an example from this work:

> What must be the amount of a square, which, when twenty-one dirhems are added to it, becomes equal to the equivalent of ten roots of that square? *Solution*: Halve the number of the roots; the moiety is five. Multiply this by itself; the product is twenty-five. Subtract from this the twenty-one which are connected with the square; the remainder is four. Extract its root; it is two. Subtract this from the moiety of the rots, which is five; the remainder is three. This is the root of the square which you required and the square is nine. Or you may add the root to the moiety of the roots; the sum is seven; this is the root of the square which you sought for, and the square itself is forty-nine.

In modern notation the statement of the problem and its solution is:

$$x^2 + 21 = 10x$$

Solution:
$$\begin{aligned} x &= 10/2 \pm \surd[(10/2)2 - 21] \\ &= 5 \pm \surd(25 - 21) \\ &= 5 \pm \surd 4 \\ &= 5 \pm 2 \\ &= 7, 3 \end{aligned}$$

Even if the reader can not follow the mathematical notation, it should be apparent that the version written using symbols is potentially much easier to comprehend. It is enormously more compact. This compactness and its consequences for intelligibility were commented on a long time ago. William Oughtred (quoted by Cajori), an English mathematician promoting his own work in 1647 noted:

> ... Which treatise being not written in the usuall synthetical manner, nor with verbous expressions, but in the inventive way of Analitice, and with symboles or notes of things instead of words, seemed unto many very hard; though indeed it was but their owne diffidence, being scared by the newness of the delivery; and not any difficulty in it selfe. For this specious and symbolicall manner, neither racketh the memory with multiplicity of words, nor chargeth the phantasie with comparing and laying things together; but plainly presenteth to the eye the whole course and processe of every operation and argumentation.

The essence here is Oughtred's observation that by writing 'with symboles or notes of things instead of words' the argument is 'plainly presenteth to the eye'. The 'symboles' used by early mathematicians are dictated by what the printer had available. So in one of the earliest printed maths books, Cardan's *Ars magna* (1545, quoted in Cajori) the author contents himself with using abbreviations set in the text roman type to express unknowns. Vieta in 1591 (quoted by Cajori) uses single text roman capitals for unknowns. It was René Descartes in 1637 who finally established the use of lower case italic letters for unknowns. He also started the useful distinction of using letters near the beginning of the alphabet for unknown constants and letters at the end of the alphabet for unknown variables. Significantly by this date it was reasonable to expect a printer to have matching roman and italic types that could be set together. Mathematical setting is notorious for the diversity of sorts it exploits. In early mathematical works the choice of these sorts is limited to what the printer has. For example in early printed books on 'algebra' much use is made of a capital R with a scratched tail to denote root, a sort ordinarily deployed in liturgical work to denote 'Response' (e.g. Cardan, 1545).

One of the more amusingly documented discoveries made by an author seeking out unexploited corners of the printer's stocks is the eventual use of bold to denote vector quantities. Electromagnetic theory was undergoing rapid development in the late nineteenth century and this called for the development of notation in mathematics capable of distinguishing quantities which possessed both direction and size (vectors) from other non-directional quantities (scalars). The first attempt used greek type, but this failed because of confusion with other greek symbols. Maxwell in 1873 promoted German (Fraktur) type for the job. Oliver Heaviside (a populariser of Maxwell's work) was the one who started using bold (*Electromagnetic theory*, 1893, Ernest Benn Ltd):

> Maxwell employed German or Gothic type. This was an unfortunate choice, being itself sufficient to prejudice readers against vectorial analysis. Perhaps a few readers who were educated at a commercial academy where the writing of German letters was taught might be able to manage the German vector without much difficulty; but for others it is a work of great pains to form German letters legibly. Nor is the reading of the printed letters an easy matter. Some of them are so much alike that a close scrutiny of them with a glass is needed to distinguish them unless one is lynx-eyed. This is a fatal objection. But, irrespective of this, the flourishing ornamental character of the letters is against legibility. In fact, the German type is so thoroughly unpractical that the Germans themselves are giving it up in favour of the plain Roman characters, which he who runs may read. It is a relic of mediaeval monkery, and is quite unsuited to the present day. Besides there can be little doubt that the prevalent shortsightedness of the German nation has (in great measure) arisen from the character of the printed and written letters employed for so many generations, by inheritance and accumulation. It became racial; cultivated in youth, it was intensified in the adult, and again transmitted to posterity. German letters must go.
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> Rejecting Germans and Greeks, I formerly used ordinary Roman letters to mean the same as Maxwell's corresponding Germans. They are plain enough, of course; but, as before mentioned, are open to objection. Finally, I found salvation in **Clarendons**, and introduced the use of this kind of type so called, I believe for vectors (*Phil. Mag,.* August, 1886), and have found it thoroughly suitable. It is always in stock; it is very neat; it is perfectly legible (sometimes alarmingly so), and is suitable for use in formulae along with other types, Roman or italic, as the case may be, contrasting and also harmonising well with them.
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> Sometimes block letters have been used; but it is sufficient merely to look at a mixed formula containing them to see that they are not quite suitable.

This rather long quote illustrates several points about mathematicians and mathematics.

* Mathematicians are quite passionate about how their maths is presented
* Mathematicians do have a keen appreciation of the utility of notation
* Mathematicians have a real (if idiosyncratic) idea of the aesthetics of maths printing

It is also clear from this quote what a real influence the printer can have on advancing mathematical notation and maths and science themselves.

The beauty of mathematics

The somewhat eccentric writings of Oliver Heaviside do show that mathematicians have a very keen sense of what works in the notation they use to convey ideas. Beyond this utilitarian view of notation, mathematicians also appreciate wider principles involved in the proper written display of their work. This appreciation is linked to an appreciation of the maths itself. A mathematician will be very pleased if s/he is able to simplify an argument or equation and render the mathematical content more comprehensible. Mathematicians speak of the 'beauty' of a well-presented and succinct proof or newly found link between branches of mathematics. The quest for such 'beauty' keeps mathematicians busy trying to refine existing proofs. The proof of the Four Colour Map problem (any map can be coloured using only four colours without the same colour adjoining itself) reported a few years ago is acclaimed, but relying as it does on thousands of hours of computer calculations, it is regarded as very inelegant. There are many mathematicians busy trying to simplify it. The writing of equations that are compact and convey meaning easily is central to 'beautiful' mathematics.

The written language of mathematics is easily given the attributes of compactness and beauty by mathematicians. It is not surprising to see a long and close association between mathematicians and their printers, the mathematicians exploiting all the available special sorts and skills that the printer can provide. There is such density of meaning in the choice of letter, its style (roman, bold, italic), its typeface (serif, sans serif, script, outline), its alphabet (latin, greek, hebrew), its size, its position relative to other characters (subscript, superscript), that the printer's job is very challenging. The challenge is to follow the very detailed requirements of the mathematical author without any understanding of the content.

The curious thing from a typographer's point of view is how little a typographer can contribute once the mathematician has made all the choices necessary to convey the mathematical meaning. It is not too controversial to suggest that the mathematician is in fact his/her own typographer, at least in the matter of writing equations.

Maths = typography?

If mathematicians define the typography of the equations they write and 'beautiful' mathematics is well-presented equations whose meaning shines through the density of notation that they bear, then perhaps mathematics is just a highly refined typographical game? Going back to Stanley Morison's definition of typography as 'so controlling the type as to aid to the maximum the reader's comprehension of the text', then that is exactly what a mathematician does. Given that mathematics' only physical reality (its only props) are written equations, then perhaps maths really is typography.

Modern mathematical typography

Mathematical typesetting has always challenged the printer (Smith: *The printer's grammar* 1755):

> Gentlemen [authors] should be very exact in their Copy, and Compositors as careful in folowing it, that no alterations may ensue after it is composed; since changing and altering work of this nature is more troublesome to a Compositor than can be imagined by one that has no tolerable knowledge of Printing. Hence it is, that very few Compositors are fond of Algebra, and rather chuse to be employed on plain work, tho' less profitable to them than the former; because it is disagreeable and injures the habit of an expeditious Compositor.

It is not only challenging, it is also expensive. In the 1970s and 1980s publishers sought cheaper alternatives to hot-metal typesetting and used strike-on systems (IBM, Varityper) extensively. Mathematicians grumbled, but mostly accepted arguments about costs. One however was so appalled by the standards of typesetting from Varitypers that he rebelled, spent time studying typography and typesetting, exploited the just-available raster-scan typesetters, and wrote his own type design and typesetting programs. That is Donald Knuth, Professor of Computer Science at Stanford. His type design program is Metafont and the typesetting program is TeX. This is not the place to go into the workings of TeX, but it is interesting to remark that the program is a very good example of what computer scientists call an 'expert system' that is modelled to some degree on the workings of the Monotype sytem that it replaces. Typographers hated it because of its associated typeface, Computer Modern Roman which Knuth modelled on the maths books of his youth (set in Modern Series 7). Typesetters couldn't understand its input coding: this was not modelled on the mechanical requirements of the Monotype system, but is written to make sense to mathematicians. The program was also free. It has been hugely successful to the point that it is the only word-processing system any self-respecting mathematician will use. Typesetters have then found the economic necessity of dealing with it. Several are able to do something about its associated typeface, which pleases typographers.

One significant aspect of TeX that is illustrative of the theme of this paper is a little-remarked feature of it. Knuth wrote an output program abbreviated to 'dvi' which allows TeX to be printed on anything from a 64-pin dot-matrix printer to a laser raster typesetting machine. In all cases the relative positioning of the characters of the equations are perfectly maintained irrespective of the printer used. A mathematician can email a dvi file to a colleague on the other side of the world know exactly what that colleague will see ('dvi' stands for device independent). The exact arrangement of characters is vital to the meaning of the equations. The exact *typography* is vital. For Knuth *typography is maths.*

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